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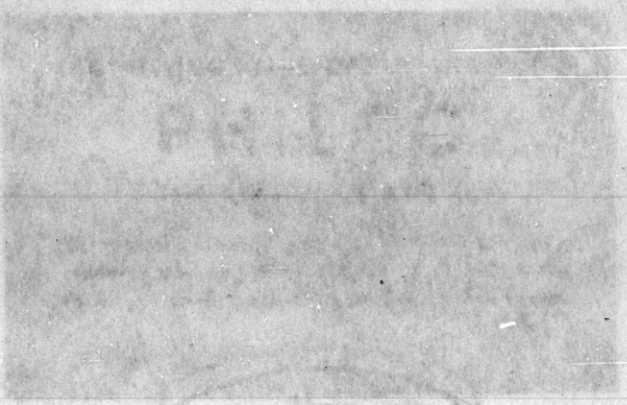

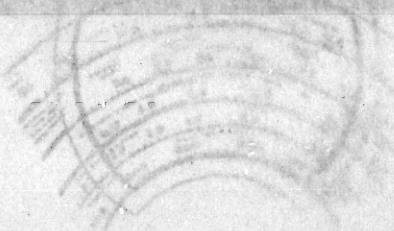

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Vol. 240 OCTOBER, 1945 No. 4

A NEW TYPE OF DIFFERENTIAL ANALYZER.

BY

V. BUSH, Eng.D., Sc.D., AND S. H. CALDWELL, Sc.D.

1.0 The differential analyzer is a machine primarily for evaluating the solutions of ordinary differential equations. It has thus been used for the solution of problems in many branches of science and engineering. But the power of the differential analyzer has been effective beyond the point of mere application to existing differential equations. By providing an economical means for producing numerical solutions of differential equations, it has stimulated activity in fields of mathematical effort where the processes of analysis are inadequate.

This paper presents a new type of differential analyzer, of greater precision, scope, and flexibility than earlier machines, but one which retains and emphasizes features of proven merit from the original design. It is intended to serve investigators working either locally or at a distance with equal ease. Hence, before presenting it in detail, the history of differential analyzers will be briefly reviewed, and then a somewhat general treatment will be given of the nature of a differential analyzer and its method of use.

2.0 The machine on which the basic principle of the differential analyzer was developed was placed in operation at the Massachusetts Institute of Technology in 1925,[1,2] and the first comprehensive differential analyzer was introduced in 1930.[3] Subsequently, the early papers of Lord Kelvin were found in which he described a method for using a machine of this type.[4]

[1] Bush, V., Gage, F. D., and Stewart, H. R., "A Continuous Integraph," *Jour. Frank. Inst.*, 208: 63–84, 1927.

[2] Bush, V., and Hazen, H. L., "Integraph Solution of Differential Equations," *Jour. Frank. Inst.*, 208: 575–615, 1927.

[3] Bush, V., "The Differential Analyzer. A New Machine for Solving Differential Equations," *Jour. Frank. Inst.*, 212: 447–488, 1931.

[4] Sir William Thomson (Lord Kelvin)—a series of papers, including one by his brother James Thomson, published in *Proceedings of the Royal Society*, Vol. 24, 262–275, Feb. 1876.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

At least five differential analyzers have been built by other organizations following the general scheme of design used in the 1930 machine.[5,6,7,8] Several machines of light weight have also been built, based on a design by D. R. Hartree and A. Porter.[9,10,11] There is one reference to a differential analyzer built in Germany,[12] but no description of the actual construction is given. Svein Rosseland refers to a machine of this type built in Leningrad, but again no description is given (see footnote 2, page 730, reference 6).

The rapid acceptance and application of the differential analyzer on an international scale demonstrated that it would be justifiable to attempt to produce one such device having higher precision, an increased capacity for handling complicated problems, and the increased flexibility necessary to serve a more extended scientific community. Accordingly, beginning in 1935, a program involving completely new development and design of the entire machine and its components was begun at the Massachusetts Institute of Technology, and this was followed by a construction program of about five years' duration. The resulting machine was placed in active service in 1942 and has subsequently carried a heavy war-time load. Further development and refinement have continued during this period and will probably continue for many years. But in all essential respects this new differential analyzer is a completed and proven instrument. It is confidently expected that it will encourage and expedite the application of mathematics in many fields of science not only at the Massachusetts Institute of Technology but at other research centers as well. It is being operated on a basis which renders it available to research workers everywhere.

[5] In 1933 a design program was undertaken jointly among the Ordnance Department of the U. S. Army, the Moore School of the University of Pennsylvania, and the Massachusetts Institute of Technology. Identical designs were thus prepared for both the Ordnance Department and the Moore School. The machine for the Ordnance Department was built by commercial shops under M.I.T. supervision and was finally assembled at the Ballistics Research Laboratory, Aberdeen Proving Center. The machine at the Moore School was built as a project of the Civil Works Administration. Both machines were completed in 1935 and were substantially identical. See Travis, Irven, "Differential Analyzer Eliminates Brain Fag," *Machine Design*, 15–18, July 1935.

[6] Hartree, D. R., "The Differential Analyser," *Nature*, 135: 940, June 8, 1935. An earlier announcement appears in *Nature*, 135: 535, April 6, 1935.

[7] Rosseland, Svein, "Mechanische Integration von Differentialgleichungen," *Die Naturwissenschaften*, 27 Jahrg., Heft 44, 729–735, 1939.

[8] Kuehni, H. P., and Peterson, H. A., "A New Differential Analyzer," *Trans. A. I. E. E.*, 63: 221–228, May 1944.

[9] Hartree, D. R., and Porter, A., "The Construction of a Model Differential Analyser," *Mem. and Proc. Manchester Lit. and Phil. Soc.*, 79: 51–72, July 1935.

[10] Massey, H. S. W., Wylie, J., and Buckingham, R. A., "A Small Scale Differential Analyser: Its Construction and Operation," *Proc. Roy. Irish Acad.*, 45: 1–21, 1938.

[11] Lennard-Jones, J. E., Wilkes, M. V., and Bratt, J. B., "The Design of a Small Differential Analyser," *Proc. Camb. Phil. Soc.*, 35 (III): 485–493, July 1939.

[12] Sauer, R., and Pösch, H., "Integriermaschine für gewöhnliche Differentialgleichungen," *VDI-Zeit.*, 87: 221–224, April 1943.

NATURE OF THE MACHINE.

3.0 There are several possible types of machines for evaluating the solutions of ordinary differential equations. Except for those involving step-by-step procedures they all operate by constructing or arranging a physical system so that its performance is governed by the equation to be solved, and then recording the performance.

The differential analyzer is such a machine in which the values of the variables involved are represented by the positions of rotating shafts. These shafts are interconnected by mathematical units which force them to move in accordance with the functional relations expressed in the equation.

Only a few types of the basic mathematical units are required, but a relatively large number of identical units of each type is provided. These are arranged with respect to a versatile "switchboard" so that the working units can be interconnected in whatever combinations are demanded by the problems to be solved.

This section will discuss the characteristics of the various components of the machine and their relations to the operations which must be performed in producing solutions of differential equations.

3.1 A differential equation is an equation which contains functional relationships between variables in the form of derivatives or integrals. Its general form may be written

$$f\left[\frac{d^n y}{dx^n}, \frac{d^{n-1}y}{dx^{n-1}}, \cdots, \frac{dy}{dx}, y, x\right] = 0. \tag{1}$$

Usually it is possible to isolate the highest-order derivative present * and express the equation in the form

$$\frac{d^n y}{dx^n} = f_1\left[\frac{d^{n-1}y}{dx^{n-1}}, \cdots, \frac{dy}{dx}, y, x\right]. \tag{2}$$

For example, the differential equation

$$\frac{1}{y^2}\frac{d^3y}{dx^3} - \left[2\frac{d^2y}{dx^2} - \frac{1}{y}\sin\frac{dy}{dx}\right]^2 = 0$$

is expressed in the form given by eq. (1). It can readily be rearranged to

$$\frac{d^3y}{dx^3} = \left(2y\frac{d^2y}{dx^2} - \sin\frac{dy}{dx}\right)^2, \tag{3}$$

in which form it corresponds to eq. (2).

This simple step of isolating the highest-order derivative is a basic one in mechanizing the solution. All the lower-order derivatives and the dependent variable can be obtained by integration from the highest-

* In transcendental types of equations this may not be true. See Section 4.8 for procedure in such cases.

order derivative. But when the equation is written in the form of eq. (3), it becomes evident that the highest-order derivative is itself a function of the lower-order terms.

It is necessary first to break down an equation according to the basic operations which must be performed and then to assemble an analogue of it, using mechanisms which perform such operations. Equation (3) will be a useful example for illustrating some of these steps.

3.2 The basic means for representing the value of any variable on the machine is the angle through which a rotating shaft turns, starting from any known initial position, and with one direction of rotation chosen to represent the positive direction of variation.

Each shaft representing a variable is assigned a numerical scale factor * which specifies the number of revolutions of that shaft which equals one unit of the variable represented. Any particular variable may appear in more than one place in the machine assembly if the variable is required at different scale factors for different purposes.

It should be emphasized that the instantaneous value of any variable is always represented by the angle through which its shaft has turned at that instant, and is never related to the speed of the shaft. The speed of operation determines the time required for solving an equation but it does not influence the numerical results obtained except through the deterioration of unit performance because of very high or very low speeds.

3.3 Spur gears are used to make changes in scale factors and to introduce numerical coefficients, such as the "2" in eq. (3). To provide for all possible needs such gears must be supplied liberally and designed so that ratios can readily be changed.

In the new differential analyzer, gearing is provided in the form of adjustable gear clusters, designated as decimal gear boxes. There are two types: The first is a one-digit type which can be set for any ratio between 0.1 and 1.0, in steps of 0.1; the second is a decade decimal box which will provide any ratio between 0 and 1.1110 in steps of 0.0001. The one-digit type is adequate for the gross adjustments of scale factor which are frequently made, while the decade type is useful in introducing coefficients and fixed parameters to four significant figures.

Remote control of the setting of all ratios is provided. This feature is discussed as part of the general design of the gear boxes in section 6.4.

3.4 In the example of eq. (3) it is necessary to perform the process of subtraction. Both addition and subtraction are performed by planetary differential gears. The relative directions of rotation of the input shafts determine whether the gearing adds or subtracts. The process is a continuous one, and the output motion of the differential gear, or "adder," is the instantaneous sum or difference of the input variables.

* The computation of scale factors is discussed in Appendix IV.

It is usually necessary to insert a gear ratio between a variable and an adding gear in order to equalize the scale factors of the terms to be added or subtracted. For this reason the adding gears of the new machine are always associated with decimal gear boxes. Substantial savings in space and equipment are effected by the design described in section 6.4.

3.5 The example of eq. (3) contains no explicit requirement for the process of integration. It is somewhat paradoxical that the most important element of any differential analyzer is the integrator. Integrators are used in order to produce lower-order derivatives from higher-order derivatives, in accordance with the defining equation

$$\frac{d^{n-1}y}{dx^{n-1}} = \int \frac{d^n y}{dx^n}\, dx.$$

But it is also true that an equally useful relation is that

$$\frac{d^n y}{dx^n} = \frac{d}{dx}\left[\frac{d^{n-1}y}{dx^{n-1}}\right],$$

and, on paper at least, the solution of differential equations can be mechanized using either of these relations. The choice of using the integrator rather than the differentiator is governed entirely by the fact that there is no satisfactory differentiator available.*

The integrator is a mechanism having two input shafts and one output shaft, arranged so that the ratio of an incremental rotation of the output shaft to an incremental rotation of one of the input shafts is a continuous linear function of the angular position of the second input shaft. Thus, if

W = angular position of the output shaft,
V = angular position of one input shaft,
U = angular position of second input shaft, and
k = a constant of the integrator,

the integrator mechanism relates these quantities in accordance with the expression

$$\frac{\Delta W}{\Delta V} = kU. \tag{4}$$

At any instant this expression takes the derivative form

$$\frac{dW}{dV} = kU,$$

* The process of integration can be made highly accurate, for it involves essentially the successive additions of quantities of nearly like size. Differentiation, however, requires essentially the subtraction of nearly like quantities. It follows that the performance of differentiation by mechanical means involves an inherent difficulty which renders great precision of operation unattainable.

from which is obtained the equation of the integrator

$$W = k\int_{V_1}^{V} U dV. \tag{5}$$

In the above derivation no restrictions were placed on the natures of the input variables U and V. The integrator performs its basic task for any inputs of variables which can be represented by the rotations of shafts.* A number of integrators are available in the differential analyzer, and they can be applied freely in performing integrations, without regard to analytic difficulty.

The geometric structure of the integrator used is illustrated in Fig. 1, in which the input motion V rotates a flat disc D about a vertical axis. A sharp-edged wheel T rests on the disc and is driven by friction so as to produce the output rotation W when the disc is turned. The input motion U is applied to the lead screw S, which in turn moves the nut N carrying the output wheel T. Thus the distance d of the wheel from the center of rotation of the disc is varied in proportion to U. For this structure it is evident that eq. (4) holds.

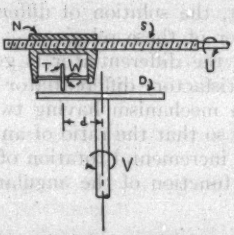


FIG. 1. Geometry of the integrator.

Figure 1 does not show how the input motions U and V are supplied to the structure, nor does it show how the output motion W is made to do useful work. The actual construction details are given in section 6.3. For the present it will be assumed that when inputs U and V are supplied, an output W is obtained, capable of driving other equipment,† and that these motions are related by eq. (5).

3.6 Referring again to the example of eq. (3), there are two functional operations which must be performed. The sine of one variable

* In the derivation of the integrator equation all variables were defined in terms of the angular positions of their respective shafts. The "rotation" or "motion" of any shaft are terms used to designate a succession of angular positions each of which represents an instantaneous value of a variable.

† It should be noted that as actually constructed the integrator is not mechanically reversible, that is, one cannot apply a motion W and obtain the motion V by friction drive. An inverse connection of the integrator is possible, as described in Appendix II.

must be produced, and the entire term on the right-hand side of the equation must be squared.

These are typical of the common functions demanded by various differential equations. The differential analyzer must contain equipment for producing any required function of any variable appearing in a problem. Actually, the new machine is quite versatile in its ability to provide functional relationships. The various methods and equipments used are described in sections 5.0 to 5.4, and 6.5.

3.7 Multiplication of variables is carried out in two ways. The first of these mechanizes the familiar relation

$$P \cdot Q = \int P dQ + \int Q dP.$$

The machine is well supplied with integrators and adding gears. Hence it is merely necessary to feed the motions P and Q to two integrators; to the U and V shafts, respectively, of one, and to the V and U shafts, respectively, of the other; add the outputs of the two integrators by means of an adding gear, and the result is the desired product $P \cdot Q$.

Sometimes the product to be performed occurs under an integral sign, such as $\int P \cdot Q d\eta$, or can be put in that form. The integral of a product is produced by two integrators by making use of either of the identities

$$\int P \cdot Q d\eta \equiv \int P d\left[\int Q d\eta\right] \equiv \int Q d\left[\int P d\eta\right]. \qquad (6)$$

For example, η is applied to the V-shaft of an integrator and Q to its U-shaft. The output W of the integrator is $\int Q d\eta$. This integral is now fed to the V-shaft of a second integrator and P is applied to the U-shaft. The second unit integrates P with respect to the variable of integration, $\int Q d\eta$. Since

$$d\left[\int Q d\eta\right] = Q d\eta,$$

the output of the second integrator is the required integral of eq. (6).

In order to divide one variable by another it is usual on the new machine first to form the reciprocal of the denominator and then use the above procedure to multiply this reciprocal by the numerator. Any of the various methods for introducing functions can be used in obtaining the required reciprocal.

Where division occurs within an integral, the reciprocal of the denominator may be obtained and treated as one term of the integral of a product, as discussed under eq. (6). It is also possible to use an integrator, according to a method described in Appendix II, so that it

evaluates

$$V = \frac{1}{k}\int \frac{dW}{U}, \qquad (7)$$

instead of the normal output of eq. (5). An integrator so operated can be used to obtain the integral of a quotient, for example, $\int \frac{P}{Q} d\eta$. First use an inverse integrator to obtain $\int \frac{d\eta}{Q}$ according to eq. (7). Then use a normal integrator by the method of eq. (6) to produce

$$\int P d\left[\int \frac{d\eta}{Q}\right] = \int \frac{P}{Q} d\eta.$$

3.8 It has been pointed out that much of the procedure for solving a differential equation consists of numerous applications of the basic operations of addition, subtraction, multiplication, division, integration, and the production of functions. The differential analyzer contains a supply of mechanisms designed to perform these tasks, along with means for setting up the required interconnections among individual units. The next section will deal with the technique of interconnection.

TECHNIQUE OF THE MACHINE APPROACH.

4.0 The classification of differential equations, and much of the terminology regarding them, has evolved from the analytical treatment to which they have been subjected. Much of this is quite at variance with the machine approach to the problem.

The outstanding characteristic of the machine approach is its logical simplicity. By analytic methods there is no single route which leads to the solution of differential equations generally. The machine approach, on the other hand, includes a basic method which applies to all ordinary differential equations.

Two principles enable us to set up the machine to handle any differential equation. These are:

1. By use of a few types of mathematical units, shaft rotations can be produced to represent each term in the equation.

2. The individual units can be interconnected so that the grouping of terms and the equality among the terms demanded by the equation are satisfied.

The interconnection of the units in a differential-analyzer assembly is most conveniently discussed with the aid of diagrams and special symbols. The general nature of an interconnection will be evident, but reference to Fig. 2 will identify the symbols corresponding to the mathematical units used. Only the basic aspects of interconnections

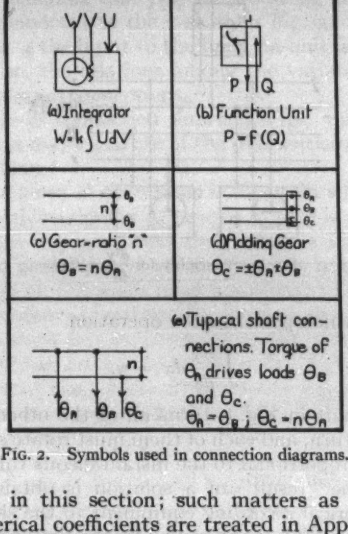

(a) Integrator
W = k ∫ U dV
P
Q
(b) Function Unit
P = F(Q)
n
(c) Gear-ratio "n"
θB = n θA
(d) Adding Gear
θC = ± θA ± θB
(e) Typical shaft connections. Torque of θA drives loads θB and θC.
θA = θB ; θC = n θA

FIG. 2. Symbols used in connection diagrams.

will be discussed in this section; such matters as gear ratios, scale factors, and numerical coefficients are treated in Appendix IV.

4.1 A solution on the differential analyzer logically starts with the assignment of a shaft to represent the independent variable. This may be one of the longitudinal "bus" shafts of the older-type machine, or the output of an independent-variable unit (see section 6.2) in the new machine. In either case the shaft motion is created by a drive motor operating at any reasonable (not necessarily constant) speed, and this motion is the datum to which all other shaft motions are related.

4.2 An example of a completely straightforward type of interconnection is given by the solution of

$$\frac{d^2y}{dx^2} = f(x), \tag{8}$$

as shown in Fig. 3. The rotation of the x-shaft feeds continuously into a function unit, which in turn drives the shaft marked $f(x)$. Since $f(x)$ is, according to the equation, equal to d^2y/dx^2, its shaft is connected to the integrand input of integrator I, and the x-shaft is connected to the variable-of-integration input of the same integrator. These two inputs are continuous, and the integrator supplies at its output shaft

$$\int \frac{d^2y}{dx^2}\,dx = \frac{dy}{dx}.$$

Similarly, the dy/dx-shaft and the x-shaft are connected to the inputs

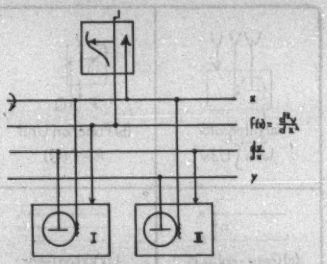


FIG. 3. Assembly for $\frac{d^2y}{dx^2} = f(x)$.

of integrator II, which performs the operation

$$\int \frac{dy}{dx}\,dx = y.$$

When the x-shaft in Fig. 3 turns all of the other shafts in the assembly must also turn, and each of them must rotate so that its position at all instants is proportional to the instantaneous value of the variable it represents. The "result" of a solution is obtained by attaching graphical or numerical recording equipment to the shafts which represent the required variables, and recording the simultaneous positions of those shafts.

4.2 There is no particular need for using a differential analyzer to solve the preceding example; it is merely a matter of ordinary integration. But a very simple change, to the following equation

$$\frac{d^2y}{dx^2} = f(y), \tag{9}$$

may radically increase the difficulty of solution by analysis. On the differential analyzer only the change of a single connection is required, as shown in Fig. 4.

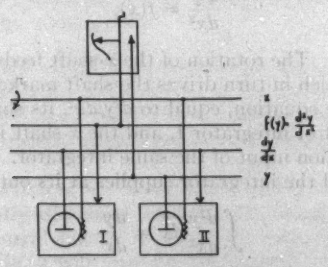


FIG. 4. Assembly for $\frac{d^2y}{dx^2} = f(y)$.

Equation (8) demands that $f(x)$ be produced, so the input to the function unit is connected to the x-shaft in Fig. 3. In eq. (9), $f(y)$ is required, so in Fig. 4 the input to the function unit is transferred to the y-shaft. Otherwise, the relations among the variables are established by identically the same connections.

4.3 The change of connection shown in Figs. 3 and 4 illustrates in its simplest terms a major feature of the differential-analyzer attack on differential equations.

In Fig. 3 a sequence of operations is set up in which each new step makes use of variables already generated. This is not true in Fig. 4. The operations are the same, and they occur in identically the same sequence, but the generation of d^2y/dx^2 depends upon knowledge of y, and the generation of y depends upon knowledge of d^2y/dx^2.

This interdependence is satisfied by the type of connection shown in Fig. 4, where the shaft motion produced as the result of a sequence of operations is connected back into the sequence, a process called "feedback." Once the feedback connection is made the sequence becomes a closed system which is activated as a whole by motion of the independent-variable shaft.

It is interesting to observe that it is the feedback connection which "mechanizes" the equal sign in the equation, because it applies the constraint which forces the machine to operate so as to equalize the two sides of the equation.

No ambiguity of shaft rotation or locking of the shafts occurs as a result of the feedback connection. The relative motions of the shafts are determined by the mathematical operations performed by the mechanical units which interconnect them. Whether or not a given shaft rotates at all is solely a question of whether or not it is driven by a source of torque adequate to supply the loads imposed on the shaft.

These considerations lead to the rule of torques, which may be stated: A differential-analyzer assembly is complete and operable when all the mathematical relations required by the equation have been established among the variables represented, and when every operating shaft is connected to one and only one source of torque.

Every unit in the machine has one or more inputs which must be driven from some external sources of torque. But when these inputs are so driven, the output shaft of each unit becomes a source of torque which in turn can drive other units. The design of each unit must be such that the torque available from its output shaft is adequate to drive any required load.

The rule of torques is useful for verifying the soundness of any differential-analyzer assembly. It may be applied to Fig. 4 as follows: The x-shaft is driven by the independent-variable motor, and in turn drives the discs of two integrators. The $f(y)$-shaft is driven by the output of a function unit, and in turn drives the lead screw of integrator

I. The dy/dx-shaft is driven by the output of integrator I, and drives the lead screw of integrator II. The y-shaft is driven by the output of integrator II and feeds back to drive the input of the function unit.

It has already been shown that the connections established in Fig. 4 satisfy the equation. The application of the rule of torques shows that every shaft derives its motion from one, and only one, source of torque. Consequently, when the x-shaft is turned, all other shafts must so move that their positions represent at all instants the values of the several variables they represent.

4.4 The feedback technique shown in Fig. 4 was developed by the indirect method of modifying the more simple case of Fig. 3. A direct application of the machine method to a more typical problem may be shown by consideration of the equation

$$\frac{d^2y}{dx^2} - (\sin x)\frac{dy}{dx} + xy = 0.$$

First the equation is rewritten so as to isolate the highest-order derivative, as follows:

$$\frac{d^2y}{dx^2} = (\sin x)\frac{dy}{dx} - xy.$$

Examination of this equation indicates that two multiplying operations are required to produce the terms on the right-hand side. As shown in section 3.7 these operations are performed with less equipment if the products appear under integral signs. Accordingly, both sides of the equation are integrated, to give

$$\frac{dy}{dx} = \int (\sin x)\frac{dy}{dx}\,dx - \int xy dx.$$

The first integral can be reduced, and the equation in form for machine solution becomes

$$\frac{dy}{dx} = \int \sin x dy - \int xy dx.$$

Figure 5(a) shows the first steps in preparing the machine assembly for this equation. It starts with the conventional assignment of a shaft to represent x, the independent variable. A function unit is driven by the x-shaft, and in turn drives the shaft for $\sin x$. But at that point, progress seems to halt. A shaft is shown to represent y, but there is no torque to drive it.

This is the point at which it is necessary to anticipate that a feedback connection will be made, and to proceed with the design of the assembly just as though the y-shaft actually had a source of torque shown. If the differential analyzer is going to operate in accordance with the equation to be solved, then all necessary variables must be

represented by rotating shafts. Hence it is always proper to anticipate that any shaft can be used, regardless of whether its source of torque is indicated at the moment. This means, of course, that subsequent variables, generated under this anticipation, are on shafts which are also devoid of torque until the shaft requiring feedback is supplied with torque. The feedback connection, therefore, activates not only the shaft to which it is connected but all others dependent on that activation.

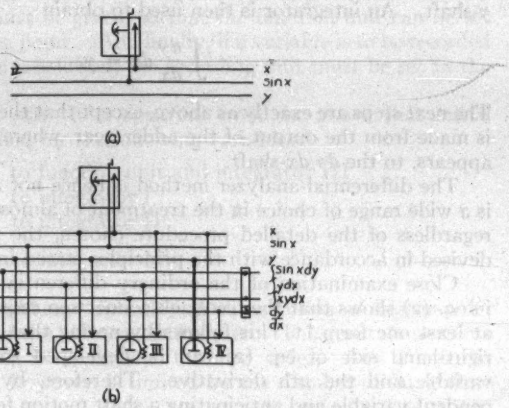


FIG. 5. Development of assembly for solving $\frac{dy}{dx} = \int \sin x dy - \int x dy dx$.

Hence, in Fig. 5(b), the y-shaft is used to supply two loads; namely, the disc drive of integrator I and the lead-screw drive of integrator II. Integrator I evaluates $\int \sin x dy$ and feeds it to one input of an adding (or subtracting) gear. Integrator II produces $\int y dx$ and feeds it to the disc drive of integrator III. The latter evaluates

$$\int x d\left(\int y dx\right) = \int x y dx,$$

and this term goes to the second input of the adder gear, with its direction of rotation opposite to that of $\int \sin x dy$. The adder gear performs the subtraction and the difference is, as defined by the equation, dy/dx. Integrator IV then carries out the integration

$$\int \frac{dy}{dx} dx = y.$$

At this point the situation is that a shaft at the top of the diagram is labelled "y" and has nothing to drive it. But by making use of that

shaft, just as though it were driven, a y-motion has been created at the output of a torque-producing unit. By merely connecting the available source of torque to the shaft for which this torque was anticipated the feedback is established, and the entire assembly is constrained to move in accordance with the differential equation. This final connection is shown by the heavy line in Fig. 5(b).

An entirely different approach may be made to the solution by anticipating the existence of torque on the dy/dx-shaft instead of on the y-shaft. An integrator is then used to obtain

$$\int \frac{dy}{dx}\,dx = y.$$

The next steps are exactly as above, except that the feedback connection is made from the output of the adder gear, where the torque for dy/dx appears, to the dy/dx-shaft.

The differential-analyzer method is hence not a rigid one, for there is a wide range of choice in the treatment of almost any problem. But regardless of the detailed procedure chosen, the machine assembly is devised in accordance with the principles stated in section 4.0.

Close examination of the ordinary differential equation as defined in eq. (2) shows that the feedback connection required always exists in at least one form.* This follows by noting that all the terms on the right-hand side of eq. (2) can be developed from the independent variable and the nth derivative. Therefore, by supplying an independent variable and anticipating a shaft motion for the nth derivative, the right-hand side of the equation can be produced, and this output can then be fed back to supply the anticipated motion of the nth-derivative shaft.

As an alternate procedure the highest-order derivative contained on the right-hand side of eq. (2) may be made the anticipated variable. From it and the dependent variable, all the remaining terms of the right-hand side are available. After forming $d^n y/dx^n$ it is integrated as many times as necessary to reach the order of the anticipated variable, where the feedback connection is made. Noting that $y = d^0y/dx^0$, this procedure is the one used in the solution shown in Fig. 5. It is illustrated also by the solution of eq. (3) shown in Fig. 11.

4.5 Once the units of a differential analyzer have been connected to solve a differential equation, with gears introduced to satisfy scale-factor requirements, the assembly is ready to operate. For each solution desired it is necessary to introduce the initial conditions of the problem.

It is a simple matter to determine the variables for which initial values must be supplied. Some of the shafts in an assembly rotate

* This does not mean that a solution always exists, but it does mean that if a solution exists, a differential analyzer can be assembled so as to obtain that solution.

without reference to any fixed location, and their incremental rotations are measured with respect to an arbitrary starting point. However, the lead-screw of an integrator moves the integrating wheel with respect to the axis of rotation of the integrating disc (see Fig. 1). Consequently, the initial value of any variable supplied to an integrator lead screw must be supplied, and before a solution begins each integrator screw must be set to its required initial value.

Likewise, the initial value of the argument of any function supplied by a function unit must be given, so that the function unit can be set for the correct starting point. And finally, if a variable is to be recorded as part of the data of a solution, the recording unit must be set to the correct initial value.

Referring to Fig. 5(b), it is apparent from the above that initial values must be supplied for the following variables:

x	to function unit and integrator III
$\sin x$	to integrator I
y	to integrator II
$\frac{dy}{dx}$	to integrator IV

4.6 There is no inherent difficulty in the solution of higher-order equations by means of the differential analyzer. The general procedure stated in section 4.4 is applicable for any order. Any particular differential analyzer, however, is limited by the number of integrators available in it.

This limitation is not of an obvious sort. It is clear that an nth-order equation uses n integrators in going from $d^n y/dx^n$ to $d^0 y/dx^0$. But it is also shown in section 3.7, and illustrated in Fig. 5, that integrators are used for multiplication. In the example of Fig. 5 four integrators are used to solve a second-order equation.

Another important use for integrators is in the generation of functions, as discussed in section 5.2 and Appendix I. This application of the integrator in many cases overshadows its use in the reduction of derivatives.

Because an integrator can perform so many duties, the number of integrators available in a given machine is not a reliable indication of the maximum order of equation which can practicably be handled on that machine. In fact the "order" of a differential equation has formal value only; it bears little direct relation to the problem of mechanization.

4.7 Simultaneous differential equations are treated on the differential analyzer just as equations of one dependent variable, except that there must be as many feedback connections as there are dependent variables. It is hence necessary in setting up the interconnections to anticipate shaft motions for a variable in each equation. Otherwise, the procedures are the same as for single equations.

Consider the simultaneous equations:

$$\frac{dx}{dt} = x + y,$$

$$\frac{dy}{dt} = \epsilon^x + y^2.$$

Figure 6 shows the connection diagram for solving these equations. The anticipated variables are x and y, and two feedback connections

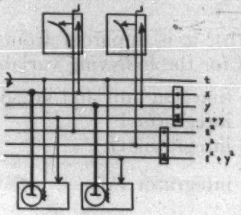


FIG. 6. Connections for the equations:

$$\frac{dx}{dt} = x + y,$$

$$\frac{dy}{dt} = \epsilon^x + y^2.$$

are indicated by heavy lines. Two function units are used to provide ϵ^x and y^2. It may readily be verified that the rule of torques is satisfied.

4.8 The transcendental differential equation is not an important practical type. Its chief interest here is that it serves admirably to demonstrate the power of the differential analyzer. As an example the following equation is manifestly difficult to handle by analytical methods:

$$\epsilon^{\frac{d^2y}{dx^2}} + \sin\left(\frac{d^2y}{dx^2} - x\right) + \frac{dy}{dx} - xy = 0.$$

Here, the highest-order derivative cannot be isolated, but the equation can be solved either in the form

$$\frac{d^2y}{dx^2} = \ln\left[-\sin\left(\frac{d^2y}{dx^2} - x\right) - \frac{dy}{dx} + xy\right],$$

or in the form

$$\frac{d^2y}{dx^2} = x + \sin^{-1}\left[-\epsilon^{\frac{d^2y}{dx^2}} - \frac{dy}{dx} + xy\right].$$

In either form the equation is solved by ordinary procedure, except that the anticipated variable must be the highest-order derivative.*

Using the second form of the equation, the machine connections are given in Fig. 7, with the feedback connection shown by a heavy line.

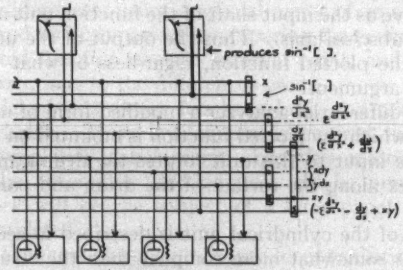


FIG. 7. Connections for the transcendental equation:

$$\frac{d^2y}{dx^2} = x + \sin^{-1}\left[-e^{\frac{d^2y}{dx^2}} - \frac{dy}{dx} + xy\right].$$

Note that in this case the product xy cannot be put under an integral sign and hence is produced explicitly from the relation

$$\int xy = \int x dy + \int y dx.$$

5.0 Frequent use of a "function unit" has been indicated in the preceding discussion. This device by definition is one which receives an input motion from any shaft which represents the argument of a function, and delivers as its output the motion of a shaft which represents the required function.

There are a number of devices and methods for introducing functions. The choice of method is determined largely by availability of equipment, considerations of precision, and convenience. Among the various devices employed three general classes may be distinguished: (*a*) manually-operated types; (*b*) automatic types; and (*c*) generating systems which require no special equipment.

5.1 One form of manually-operated function unit, called an input table, is described in reference 3. It provides a flat surface on which is mounted a large-scale plot of the required function. An index "pointer" is arranged so that it can be moved to any part of the plot by the combined action of two lead screws located at right angles to

* Isolation of the highest-order derivative, as discussed in section 3.1, is generally a useful procedure, but it need not be done. If for any reason terms involving the highest-order derivative appear on both sides of the equation, the highest-order derivative must be the anticipated variable.

each other. The lead screw which controls the abscissa motion of the index is driven by the shaft in the machine assembly which represents the argument of the function. The ordinate motion of the index is controlled by an operator turning a crank geared to the ordinate lead screw. It is the job of the operator to keep the index continuously on the plotted curve as the input shaft of the function unit moves the index parallel to the abscissa axis. Thus the output of the unit is related to the input by the plotted function, regardless of what variable in the problem is the argument.

In the new differential analyzer a modified form of manual function unit is used in which the plotted function is mounted on a drum of large diameter. The input to this unit rotates the drum, and the operator moves the index along the surface of the drum and parallel to its axis of rotation.

The design of the cylindrical unit is described in section 6.5. Mechanically, it is somewhat more compact than the structure based on cartesian co-ordinates. It is also a more convenient structure for very large scale plots. The input variable can be continued through a number of revolutions of the drum. If the output scale is so large that the plot goes off the end of the cylinder, a discontinuity can be introduced to move the curve back on the useful surface. When the operator reaches that point the machine is stopped and the index is moved to the new section of the curve without changing the position of the output shaft.

It is quite evident that any manually operated type of function unit provides a source of torque at its output shaft. The work is done by the operator who turns the crank.

5.2 Either of the above types of manual units may be converted to automatic operation. Some sort of sensing device is provided to detect the magnitude and direction of any departure of the "index" from its true position. The output of the sensing device then controls a servo mechanism to keep the index always at the proper point.

Such systems usually require special preparation of the function to be followed. One development of this type used a photoelectric method to follow a function represented by the boundary between a white and a black area.[13] In another type, which has been in use for some years, the function is prepared by cutting a spiral groove on a relatively long cylinder. A needle point rides in the groove, and deflects a light coil of wire in an appropriate magnetic circuit to detect any departure of the needle from its correct axial position as the cylinder revolves.

A radically different approach to the problem has been under development at the Massachusetts Institute of Technology as part of the main program of differential-analyzer development.

[13] Hazen, H. L., Jaeger, J. J., and Brown, G. S., "An Automatic Curve Follower," *Rev. Sci. Inst.*, 7: 353–357, September 1936.

Briefly, the basic idea is to specify the required function by a tabulated series of ordinates and their differences, up to the third or fourth differences, for uniform intervals of the argument. The ordinates and differences for a given function are prepared and stored in the form of a punched paper tape. The function is generated by integrating the differences, using a special array of small integrators. Precision is secured by automatically comparing the generated function with the tabulated function at each interval point, and feeding back any errors as corrections to the differences used.

A study model of this device has been built and tested, and it is known to be workable. More development is required, however, before the design is completely satisfactory. For this reason no detailed description of the automatic function unit will be presented in this paper. The work will be the subject of a later paper when the development is completed.

5.3 Perhaps the most elegant procedure for introducing functions is that which generates them by interconnections of other differential-analyzer components. The procedure is a simple extension of basic technique.

If a differential equation can be written which has a desired function as a solution, that equation may be treated as a definition of the function. If, furthermore, the function-defining equation does not involve the use of plotted functions in its solution, a portion of the differential-analyzer equipment may be used to solve the defining equation and thus to generate the required function.

Fortunately, many of the analytic functions can thus be produced conveniently. The use of this method has made it unnecessary to provide for so many manual and automatic function units as would otherwise be required.

A few examples will serve at this point to illustrate the procedure. Appendix I contains further discussion of generated functions.

Let the general function be $P = f(Q)$. One of the simplest cases is

$$P = Q^2.$$

For this case the defining equation, obtained by differentiation, is

$$\frac{dP}{dQ} = 2Q.$$

Figure 8 shows how a single integrator is used to solve this equation and thus to produce the square of a variable. The "independent variable" in Fig. 8 may be any variable in the main assembly for which the square is required.

For the simple exponential function

$$P = \epsilon^Q,$$

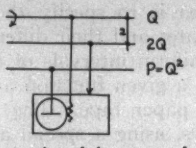


FIG. 8. Generation of the square of a variable.

differentiation gives

$$\frac{dP}{dQ} = P.$$

This function is generated by a single integrator connected as shown in Fig. 9. The negative exponential is generated in the same manner

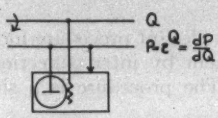


FIG. 9. Generation of the exponential.

except that a change of direction of rotation must occur between the integrator output and the lead screw. It should be noted that the rule of torques is completely satisfied by this connection. Again the input Q may be any variable which is to be the argument of an exponential function.

One of the most useful interconnections is that which produces the sine and cosine. The steps in writing the defining equation are:

$$P = \sin Q,$$

$$\frac{dP}{dQ} = \cos Q,$$

$$\frac{d^2P}{dQ^2} = -P.$$

Figure 10 shows the connections required for the last equation. Note that both the sine and cosine are produced by the connection; this is

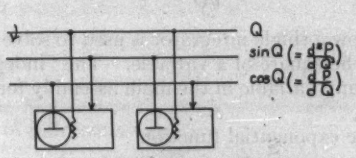


FIG. 10. Generation of sinusoidal functions.

true, regardless of whether sine or cosine is written in the first equation above. If the sign in the defining equation is positive, the hyperbolic sine and hyperbolic cosine are produced.

5.4 The generation of functions in an actual equation may be illustrated by treating eq. (3) in section 3.1. For convenience the equation is repeated:

$$\frac{d^3y}{dx^3} = \left(2y\frac{d^2y}{dx^2} - \sin\frac{dy}{dx}\right)^2.$$

An assembly which contains no function units is given in Fig. 11. Every procedure used has been illustrated in earlier examples. A somewhat different schematic arrangement is used to emphasize the

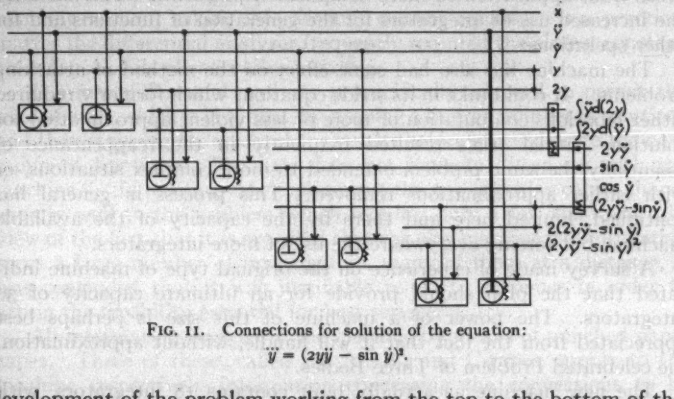


FIG. 11. Connections for solution of the equation: $\dddot{y} = (2y\ddot{y} - \sin\dot{y})^2$.

development of the problem working from the top to the bottom of the diagram. For convenience, the "dot" notation is used to represent derivatives; i.e., $\dddot{y} = d^3y/dx^3$, $\ddot{y} = d^2y/dx^2$, etc. The feedback connection is indicated by a heavy line.

DESCRIPTION OF THE NEW DIFFERENTIAL ANALYZER.

6.0 Experience gained from the operation of the original type of differential analyzer, advances made in the technique of using the machine, and the need for increased precision, were the factors which largely set the requirements to be met in the design of a new differential analyzer.

The basic objectives of development were as follows:

(*a*) Increased capacity should be provided, sufficient to meet initial operating needs, and with the overall design such as to permit expansion.

(*b*) Flexibility of operation was needed to make possible rapid shifts

from problem to problem, and to make the most effective use of the large capacity of the machine.

(*c*) Higher precision was demanded by the increased acceptance of the machine method and the consequent need for application to more critical problems.

It is evident from the preceding sections that the integrator is the most versatile unit in the machine. Therefore, it is not surprising that the capacity of a differential analyzer is measured by the number of integrators available. There must also be included, of course, a related number of units of other types as part of a balanced design.

For the most part the demand for greater capacity in the machine arose from application to more complicated problems. This included the increased use of integrators for the generation of functions and for other operations.

The machine has also had some effect on the method of attacking problems. It could take in its stride equations which formerly required either laborious computation or more or less violent approximation for solution. Initial trials resulted frequently in the reappearance of essentially the same problem extended to more complex situations, or with earlier approximations removed. This process in general has continued, limited here and there by the capacity of the available machine, but forcing everywhere the use of more integrators.

A survey made of experience on the original type of machine indicated that the plan should provide for an ultimate capacity of 30 integrators. The power of a machine of this size is perhaps best appreciated from the fact that it will handle, without approximation, the celebrated Problem of Three Bodies.

The new machine as actually built contains 18 integrators, with provision made for expansion to 30 integrators if that size becomes justified.

The very large machine involves a problem of economy of use. Many of the problems which reach it use only a fraction of its capacity. Hence it is necessary to provide for simultaneous operation of the machine on more than one problem or more than one solution at a time of the same problem. Otherwise the dead weight of unused equipment would become a serious cost burden on the small problem.

With a machine of 18 integrators it was difficult to decide whether to provide for operation in two or three sections, but the allowance for expansion to 30 integrators made three sections clearly advisable. This decision had to be made at the outset, for it affected the over-all design at many points.

The basic requirement for increased speed and flexibility called for notable reduction in the time required to interconnect the machine, principally in order that problems requiring relatively few solutions at

a time could be treated as economically as the "mass production" type. It was a disadvantage of the original type of machine that the assembly of a problem by means of an array of shafts, gears, and couplings required a substantial amount of time, frequently one or two man-days. Once the assembly was completed solutions came swiftly, and after a few days of operation the "set-up" time per solution became tolerably small and even negligible.

The situation was unsatisfactory in two respects. First, a machine of this type could not render effective aid to the user if the number of solutions required was so small that the time required to assemble the machine was comparable with the time required for numerical integration. Second, even when the amount of work justified the use of the machine, it was a rare problem that could be treated entirely by advance planning without taking into account current results. As a practical matter the differential analyzer frequently required the almost constant attendance of its client while operating.

The new differential analyzer is specifically adapted to permit the kind of work which is best done by running a limited number of solutions and then stopping for study of the results before proceeding to the next group of solutions. The user does not have to "keep up with the machine." In meeting this requirement the new machine caters almost equally well to the problem which requires only a few solutions. In view of the fact that it is expected that the new machine will ultimately serve a large number of investigators, many of them at a distance, it was concluded that it was justifiable to go to extremes in order to attain the flexibility essential for best service.

All information is given to the machine by means of punched paper tapes. Three of these, called the A-, B-, and C-tapes, supply to the machine respectively information concerning the connections to be made, the values of gear ratios in the machine assembly, and the values of initial conditions. These tapes are prepared in advance by routine procedures, using a special high-speed punch, and no differential-analyzer time is used during the process. When the punched tapes are ready, the problem is placed on the machine by automatic controls which "read" the first tape and make the specified assembly. In this new system motion is transmitted electrically from unit to unit at low energy level by means of servomechanisms. The punched tape merely controls the operation of an automatic switching system which makes the servomechanism connections. For the largest problem so far put on the machine, the entire assembly was done in 3 to 5 minutes—less than the time required for a single solution.

A similar tape, also prepared in advance, automatically sets the various gear ratios required, and a third tape puts in the initial conditions. These tapes are kept on file and it is the work of only a few minutes to put a problem back on the machine for further study.

Corresponding facility is provided for taking the results off the machine. Graphical results are available from curve-plotting output units, but in most cases the prime data are numerical tabulations of the required functions. These tabulations are prepared on automatic typewriters controlled from special counter units. Negative numbers are written directly instead of in complementary form, and additive constants, such as those required for initial conditions, are correctly applied. Usually the scale factor is a power of ten so that the typed data sheet is immediately ready for use by merely locating the decimal point.

Further details of the operational features incorporated to improve the speed and flexibility of using the machine are given in section 7.

The broad requirement for increased precision led to the specific objective of attempting to obtain over-all machine operation to within one part in ten thousand. In order to do this it became necessary to reduce critical errors in some individual components to a few parts in one hundred thousand. Many of the design features shown in the following sections originated in this requirement.

The descriptions which follow give only the salient features of the new differential analyzer. A series of papers is in preparation reporting more fully on the development of the integrator, the remotely-controlled gear box, the automatic function unit, the lead-screw setting mechanism, the high-speed counting and recording system, the basic servomechanism design, and the main automatic control system.

6.1 The mechanical switchboard, which was adequate for the original machine, could not provide the required flexibility of operation of the new machine. Consideration of various alternatives led to a decision to use servomechanisms for driving all mechanical loads, and an automatic electrical switchboard to interconnect the data transmission elements.

An attempt was made to develop a selsyn-type of data transmission device, using no iron in the rotor and very light brushes, so that it could be driven directly by an integrator wheel. This experiment ran into excessive difficulty with stray fields. In the meantime preliminary experiments showed that it was feasible to use the variation of electric fields to accomplish what the selsyn does by the variation of magnetic fields. A unit called the "angle indicator" was developed on this basis and it meets the requirements of (*a*) operation at low torque, (*b*) transmission of position data at a low energy level over standard telephone switching equipment.

Figure 12 shows the scheme of the angle indicator. Two capacitance bridges are used, each consisting of a pair of fixed condensers ($C_1 - C_1$ and $C_2 - C_2$) and a pair of variable condensers ($Cv_1 - C'v_1$ and $Cv_2 - C'v_2$). Excitation voltages, E_1 and E_2, are supplied at a frequency of about 3,000 cycles. The output voltages, e_1 and e_2, appear

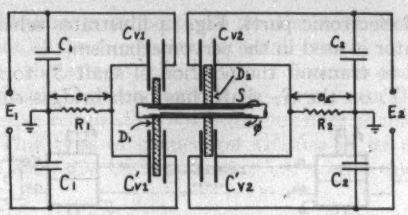


FIG. 12. Bridge circuits of the angle indicator.

across the resistors R_1 and R_2, which are actually the grid-circuit resistances of thermionic amplifiers.

The shaft S shown in Fig. 12 is any shaft in the machine which is to transmit or receive position data, represented by the angle ϕ through which it turns. Mounted on the shaft are two cam-shaped discs, D_1 and D_2, made of dielectric material, and this assembly is so located that rotation of the shaft rotates the cams between the plates of the condensers $Cv_1 - C'v_1$ and $Cv_2 - C'v_2$. Figure 13 shows the shapes of the condenser plates and a cam.

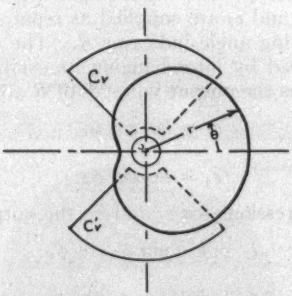


FIG. 13. Angle indicator plates and cam.

For each revolution of the shaft the capacitance of the variable condensers is thus made to vary through a cycle of values. The specific form of the variation depends upon the shape of the cam, the shapes of the fixed plates, the dielectric constant of the cam material, and the values of the associated circuit parameters. The required variation of capacitances is that which gives the results:

$$e_1 = kE_1 \sin \phi,$$

and

$$e_2 = kE_2 \cos \phi.$$

Omitting all electronic parts, Fig. 14 illustrates schematically how the angle indicator is used in the servomechanism.

It is desired to transmit the position of shaft S_T to shaft S_R. An angle indicator T, on the S_T shaft, has both bridges supplied from a

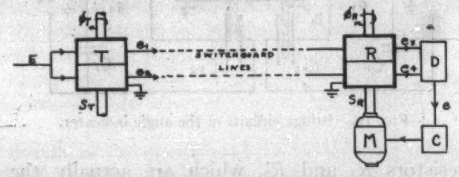


FIG. 14. Use of the angle indicator in the servomechanism.

common alternating voltage E. As S_T rotates through the angle ϕ_T the two outputs of T are:

$$e_1 = kE \sin \phi_T,$$

and

$$e_2 = kE \cos \phi_T.$$

These voltages are now transmitted over the interconnecting switchboard to the designated shaft S_R, which carries angle indicator R. The two voltages e_1 and e_2 are supplied as separate excitations to the bridges of the receiving angle indicator R. The direction of rotation of S_R may be reversed by interchanging its excitation voltages. For one set of connections the output voltages of R are

$$e_3 = ke_1 \cos \phi_R,$$

and

$$e_4 = ke_2 \sin \phi_R.$$

Substituting the expressions for e_1 and e_2, the output voltages become

$$e_3 = k^2E \sin \phi_T \cos \phi_R,$$

and

$$e_4 = k^2E \cos \phi_T \sin \phi_R.$$

These voltages are now subtracted in a transformer network D which has as its output

$$\begin{aligned} e &= e_3 - e_4, \\ &= k^2E(\sin \phi_T \cos \phi_R - \cos \phi_T \sin \phi_R), \\ &= k^2E \sin (\phi_T - \phi_R). \end{aligned}$$

If the difference $(\phi_T - \phi_R)$ is kept small by the servomechanism (this difference is designated as the error of follow-up, ϵ), the final voltage becomes

$$e = k^2E\epsilon.$$

Hence the voltage e is proportional to the difference between the angular positions of the two shafts, for small errors.

To complete the system, the voltage e becomes the input for the servo control circuit C which controls the servomotor M. The motor supplies the torque required on shaft S_R, but does so under the control of shaft S_T. This type of connection satisfies in its end results the condition that shaft S_T be the "source" of torque for the loads on shaft S_R.

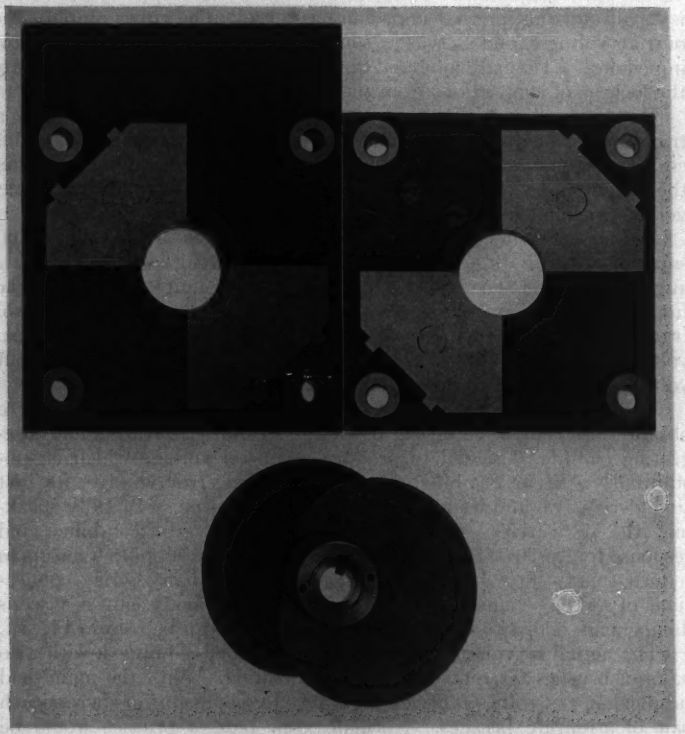


FIG. 15. Fixed plates and rotor of angle indicator.

Figure 15 shows the internal construction of the angle indicator unit, and Fig. 16 shows the completed assembly. The phase displacement between the two voltages from the unit is obtained by displacing the fixed plates of one bridge 90° away from the plates in the other

bridge. Shielding between the two bridges is provided by a grounded plate between them. In order to balance the shaft assembly, the dielectric cams are mounted 180° apart in mechanical angle. The axial displacement between the cams does not introduce appreciable vibration, even at high speeds.*

In Fig. 16 two vacuum tubes are shown mounted on the angle indicator. The resistors R_1 and R_2 of Fig. 12 are the grid input resistors for these tubes. Because the condensers used are small and the useful variations of capacitance obtained are small, the output of the bridge is small and the internal impedance is large. The tubes amplify the output voltages and act as a source of voltage with relatively low impedance. They are mounted directly on the mechanism to minimize the lengths of connections from the bridges.

A transformer network is used between the output tubes on a transmitting angle indicator and the lines to the switchboard. This circuit isolates the d-c plate circuit from the transmission line, matches the output impedance to the line, and provides adjustment of output voltage to compensate for unbalance in the bridges and tubes.

At the receiving end the transmitted voltages are amplified and supplied through transformers to the bridge circuits of the receiving angle indicator. The tubes mounted on this unit serve the same function as in the transmitter. Their outputs go directly into a pair of transformers, connected so that the voltages subtract. This final voltage, which was shown to be a measure of the error, is the input voltage to the servo controller circuits.

In the servo controller the error voltage is rectified and filtered to obtain a d-c control voltage. It then passes into derivative and integrating circuits, designed to produce a control voltage which is substantially a linear combination of the original error voltage, its first time derivative, and its integral with respect to time. Broadly speaking, the derivative voltage contributes to the servo stability and response to accelerations, while the integral voltage supplies a means to reduce steady-state lag error. Because of the many series connections of servomechanisms required in differential analyzer use, viscous dampers are added to improve stability. These can be seen in Fig. 21.

The actual servomotor is a repulsion motor † equipped with two pairs of brushes for rotation in either direction. With the main field supplied at 115 volts, 60 cycles, an induced voltage appears across each pair of armature brushes. Normally, one pair of brushes would be short-circuited for operation in one direction. For servo control, however, the brush voltages are supplied to the primary windings of two step-up transformers, and the secondary voltages become the plate

* Normal maximum speed is 2,000 rpm. The units were tested for vibration up to 7,000 rpm.

† A few special servomechanisms in the machine use d-c motors with double field windings.

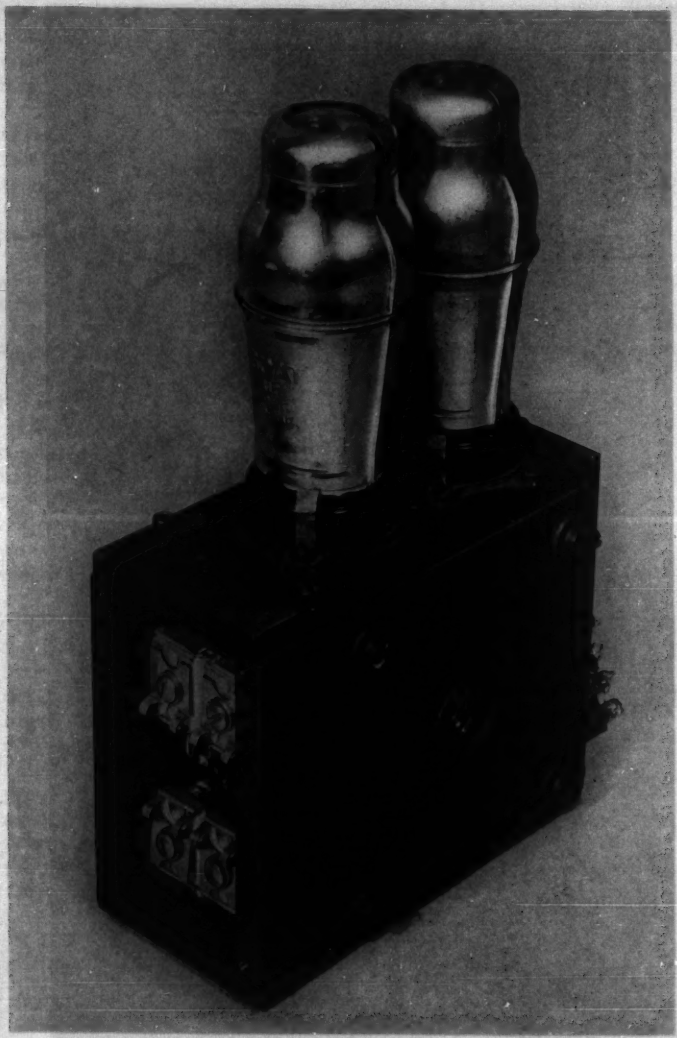


Fig. 16. Assembled angle indicator.

Rear view.

Front view.

FIG. 17. Front and rear views of servomechanism control panels. Upper panel is for receiver; lower panel is for transmitter.

voltages of a pair of thyratrons. The control voltage, combined with appropriate bias voltages, serves to cause conduction in one or the other of the thyratrons, depending on the direction of error, and the duration of conduction during one cycle is a function of the magnitude of the control voltage. Since the rotor winding of the motor supplies power to the thyratron plate circuit, the motor torque is controlled by controlling the thyratron plate current.* A typical servomotor with its angle indicator attached can be seen in Fig. 21. Figure 17 shows front and rear views of the circuit equipment panels for transmitters (small panels) and receivers (large panels).

The interconnections required in this system consist of two shielded wires (and a ground connection); these are readily handled by automatic telephone switching equipment.

6.2 Three independent-variable drives are provided so that the machine can be used if necessary in three separate sections. Any one of these units can be used with all or any part of the equipment in the machine.

Basically, the independent-variable drive is a variable-speed motor coupled to a transmitting angle indicator. This indicator can be used to furnish excitation to any required number of receiving angle indicators.† The need for such multiple connections was anticipated in the original design of the angle indicator, where it was found possible to provide a cam shape which not only gave the required output function, but also made the bridge input impedance constant.

Both the speed and acceleration of the independent variable shaft can be controlled automatically. Acceleration is controlled by loading the motor with a flywheel which is also the rotor of an eddy-current brake. This combination serves to prevent sudden changes of speed, not only during starting and stopping, but also during normal operation. Precision performance, particularly of integrators, deteriorates rapidly if excessive acceleration occurs.

Speed can be controlled manually, but automatic equipment is also provided for the purpose. Each servomotor contains a built-in a-c generator consisting of an alnico permanent-magnet rotor, and a fixed field coil. Rotation of the motor induces a voltage in the field coil proportional to speed. This voltage is rectified, amplified, and fed to the grid circuit of a gas tube. If the speed of the motor exceeds a prescribed value, the consequent voltage causes the gas tube to conduct.

In the main automatic control system of the machine provision is made so that the plate circuits of all the speed-control gas tubes of an

* The method described for the thyratron control of a repulsion motor was developed by the Sperry Gyroscope Company. See U. S. Patents No. 2,008,364 (July 16, 1935) and 2,040,014 (May 5, 1936), by F. L. Moseley.

† Experimentally there have been connections which required this transmitter to supply 11 receivers.

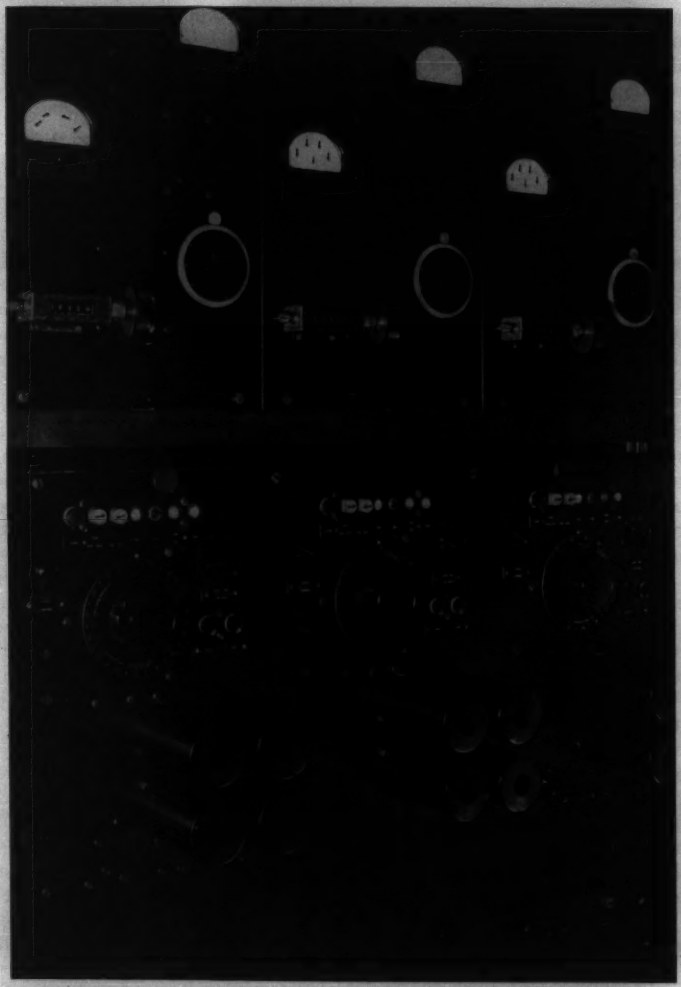


FIG. 18. Independent variable control panels.

interconnection are supplied from a common bus associated with the particular independent-variable drive being used. If any one of the gas tubes conducts, the supply bus potential drops about 100 volts. This drop of potential is used in the grid circuit of a thyratron to decrease the speed of the independent-variable motor, and the speed continues to decrease until no gas-tube conduction occurs. When the supply bus potential rises again, the thyratron permits the independent-variable speed to rise until one of the speed-control gas tube conducts, and the cycle of control begins again.

Thus, the speed of the independent variable "drifts" throughout the time it runs. This drift is not harmful because its acceleration is controlled. It has the advantage of forcing the machine automatically to seek the highest allowable operating speed, and thus automatically to minimize the time of solution.

Figure 18 shows the three sets of independent variable panels. The top panel in each set has the motor drive mounted on its rear, while on the front is a counter and dial for showing revolutions and fractions of a revolution. Each motor panel contains a speedometer and a time registering meter. The latter records the total time the independent variable operates.

Associated equipment, mounted on the bottom panel, provides for manual speed control, reversal of direction, various test functions, and automatic counter tripping circuits. The latter equipment is used to space the readings taken by the counting and recording units (see section 6.6), in terms of the independent variable. Provision is also made for manual actuation of the recorder circuits when intermediate readings are needed.

Figure 19 is a view of the rear of one of the motor panels and shows the motor, its eddy-current brake, and the transmitting angle indicator.

6.3 Integrator performance depends largely on precise construction, and the careful control of a host of dynamic factors, including speed, acceleration, vibration, the contact surface between wheel and disc, the inertia of the output wheel assembly, bearing friction, etc. A later paper will consider these factors in greater detail.

Figure 20 is the top view of an integrator with the wheel carriage removed, and Fig. 21 shows the servomechanism drives on the bottom of the same unit. The integrator disc is made of plate glass ground flat, and is cemented to a hub which is mounted directly on the shaft of its servomotor. Lead screw motion is transmitted through a precision gear train from the second servomotor. A half nut rests on the lead screw and drives the carriage. Projecting from the side of the nut is a "tail" which rides on a corrector bar attached to the side of the main frame. By tilting the corrector bar the nut is rotated through a small angle as it moves from one end of the screw to the other. This arrangement permits adjustment for inaccuracy in the pitch of the screw: a

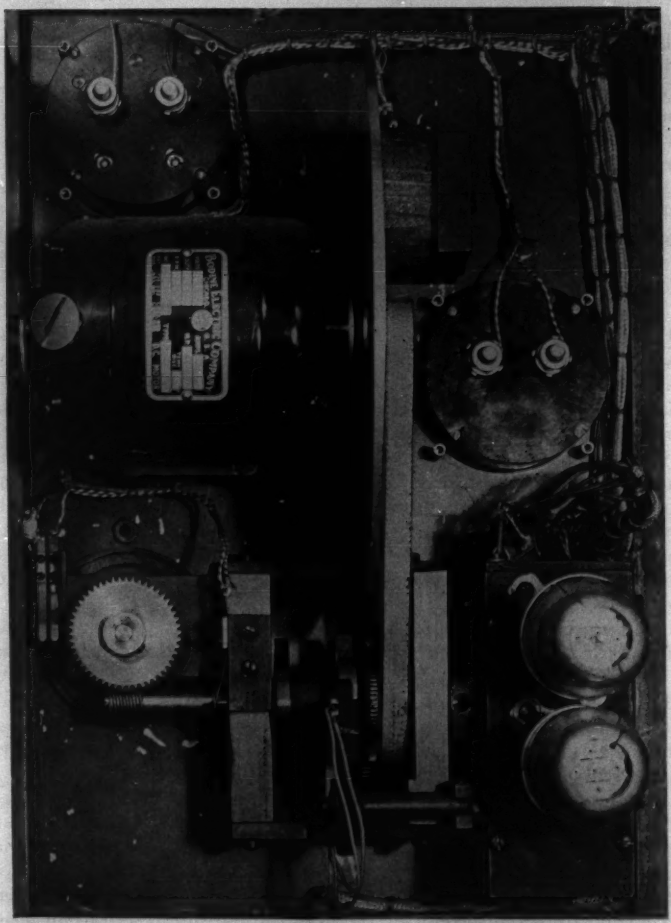


FIG. 19. Independent variable motor and angle indicator.

necessary feature because in making the screw the effort was to secure uniformity of pitch, with only approximate accuracy. The corrector bar can also be used to correct for residual errors which affect the precision of the screw, by making the edge of the bar into a cam.

Mounted on the top of the frame are three rectangular steel blocks which serve as ways for the integrator carriage. Figure 22 shows the bottom of the carriage, and Fig. 23 shows the carriage in its operating position. Three pairs of precision ball bearings are used as wheels to support and guide the carriage. One set of bearings forms a tripod for vertical support. The other set has one bearing pivoted on a short

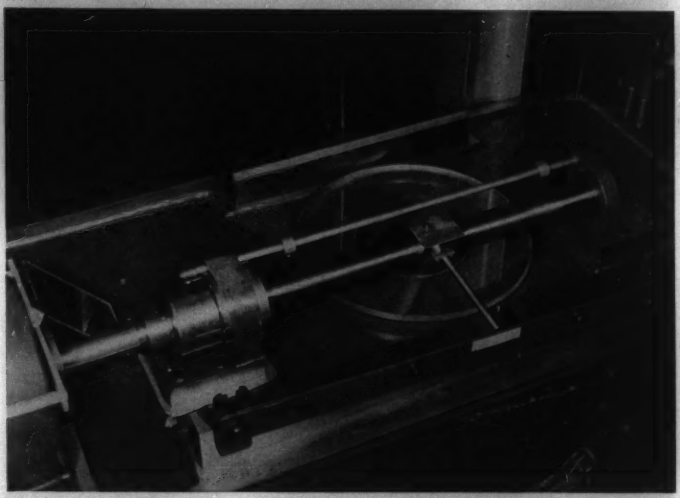


FIG. 20. Top of integrator, carriage removed.

arm, with a spring provided so as to position the carriage horizontally against the ways on one side.

The motion of the nut is transmitted to the carriage by simple pressure with no constraining link. A steel ball is pressed in one vertical end surface of the nut. This ball pushes against one face of a steel block fastened to the carriage. The entire frame of the integrator is slightly tilted so that the carriage rolls down hill against the nut.

A pivoted flat spring extends from the carriage to the top of the nut, exerts pressure to prevent the nut from rising off the screw, and also holds the corrector tail in contact with the corrector bar.

The function of the carriage is to support and accurately to locate the integrator wheel (Fig. 22). The wheel is mounted on a shaft which

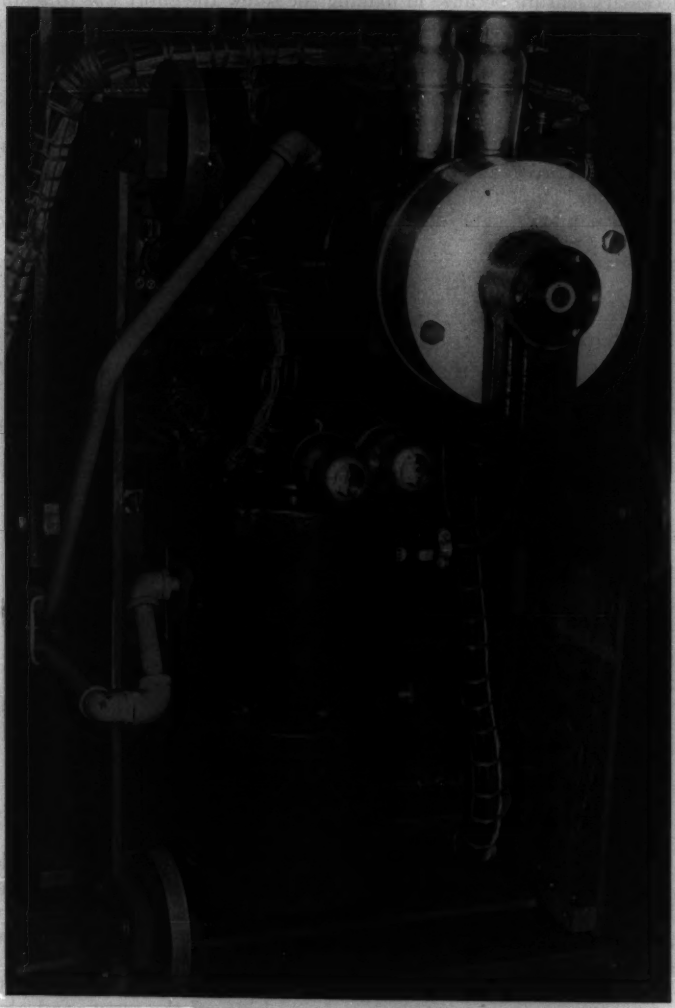


FIG. 21. Bottom of integrator, showing servomechanisms.

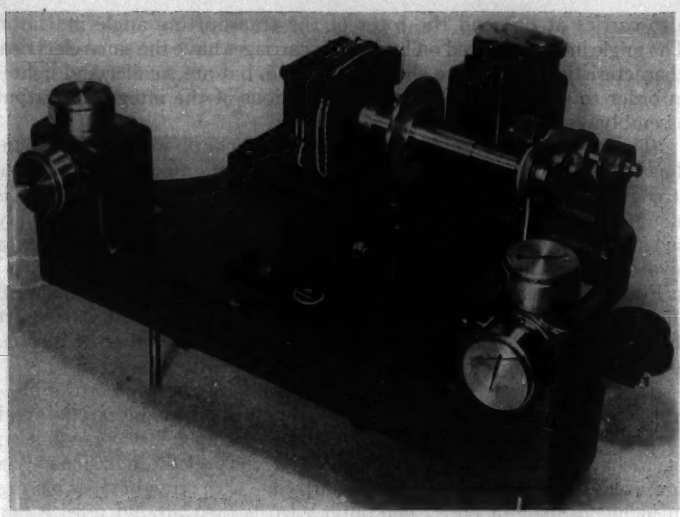


Fig. 22. Bottom of wheel carriage of integrator.

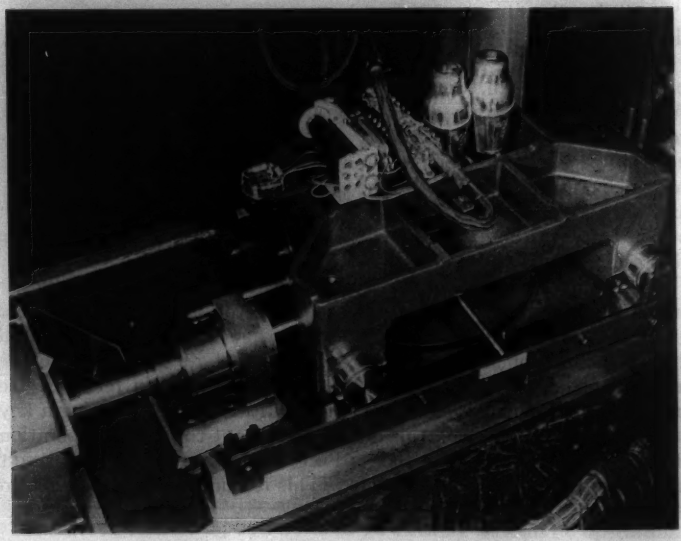


Fig. 23. Integrator with wheel carriage in place.

also carries at one end the rotor of the transmitting angle indicator. The angle indicators used on integrator carriages have the same electrical characteristics as the type shown in Fig. 16, but are smaller and lighter in order to satisfy the low torque limitations of the integrator output assembly.

The integrating wheel itself consists of a steel hub with a magnesium flange and a hardened steel tire. This construction provides adequate strength and stability with low moment of inertia. The wheel is mounted on a massive steel shaft with polished ends supported radially and axially by jewel bearings. One set of bearings is mounted in a pivoted piece which can be operated by a relay (see Fig. 22, and relay in Fig. 23) so that the wheel is raised from the disc when traversing to an initial position. The purpose of the massive shaft used to support the wheel is to obtain sufficient weight, with low moment of inertia, to provide adequate pressure of the wheel on the disc.

Investigation was made of the best profile for the edge of the integrating wheel. Substantially no difference in performance was found for a wide variety of curved and flat shapes. Hence it was possible to use a shape which could be made easily to controlled dimensions. As now used, the wheel is finished to a sharp edge with straight sides having an included angle of 60°. The outer edge is then ground until a flat surface about 0.002 inch wide is obtained. This shape can readily be made to a specified diameter, and slight variation of the width of the "edge" is allowable to meet tolerance requirements on the wheel diameter.

The integrating wheel assembly as described can deliver a torque of about 5 gram-cm. without exceeding a slip of one part in 10,000. As an extreme condition, about 4.5 gram-cm. of this torque can be used to accelerate the wheel assembly at the maximum allowable rate of 200 rpm per second. The remaining 0.5 gram-cm. provides for bearing friction.* Low friction is obtained by balancing the integrating wheel assembly so that the center of gravity is approximately above the point of contact with the disc. Thus the bearings carry only the weight of the residual unbalanced mass.

Figure 24 shows the automatic setting mechanism mounted on the end of the integrator. This device, in co-operation with an associated group of 44 relays, accepts the numerical value of the integrator initial displacement supplied by the C-tape, and sets its integrator to the required value, to a precision of one part in 150,000 in terms of the extreme displacement from center. In addition, upon operating a key on the main control panel, this device carries out the inverse operation of "reading" the position of the lead screw and transmitting the result to one of the recording typewriters. This feature makes it possible to

* Windage loss is negligibly small, even at maximum speed.

record on every data sheet the initial and final positions of all integrator lead screws.

The setting mechanism circuit also contains provision for its use in conjunction with a relay-type of multiplier-divider unit. When an

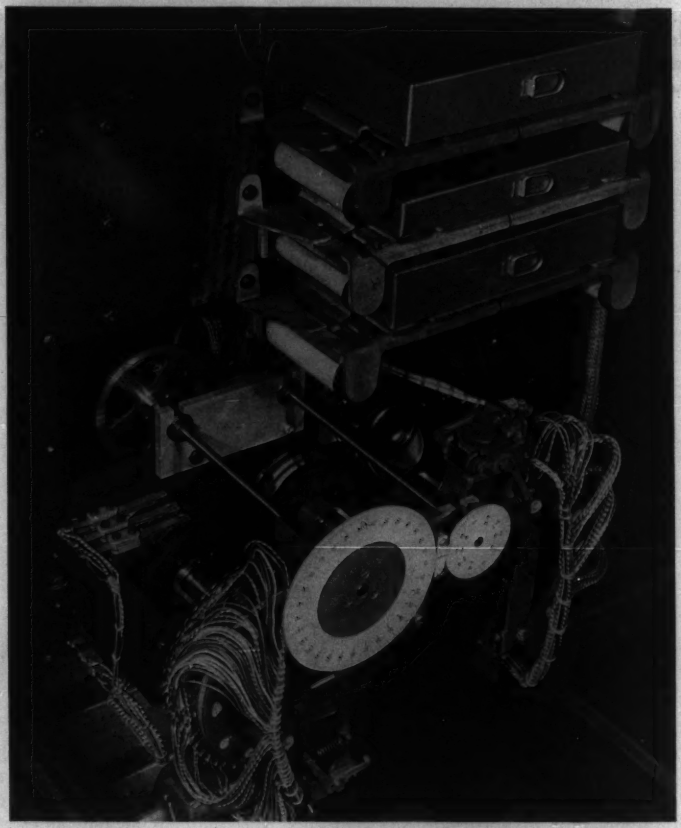


FIG. 24. Automatic setting mechanism for integrator.

integrator displacement is to be multiplied or divided by a number such as 2, 5, or 10, the setting mechanism reads the lead screw position, transmits the reading to the multiplier-divider unit, receives the new value, and sets the integrator to it in an entirely automatic operation.

This feature was introduced for its value in the solution of linear differential equations.

The setting mechanism consists essentially of a group of rotary selector switches which "hunt" for positions corresponding to the separate digits of the number to be set. These switches carry contact points to their required mechanical positions. A motor then drives the lead screw and another set of contact points until a position is found where the second set of contacts lines up with those specified by the several digits. The details of this device will be presented in a later paper.

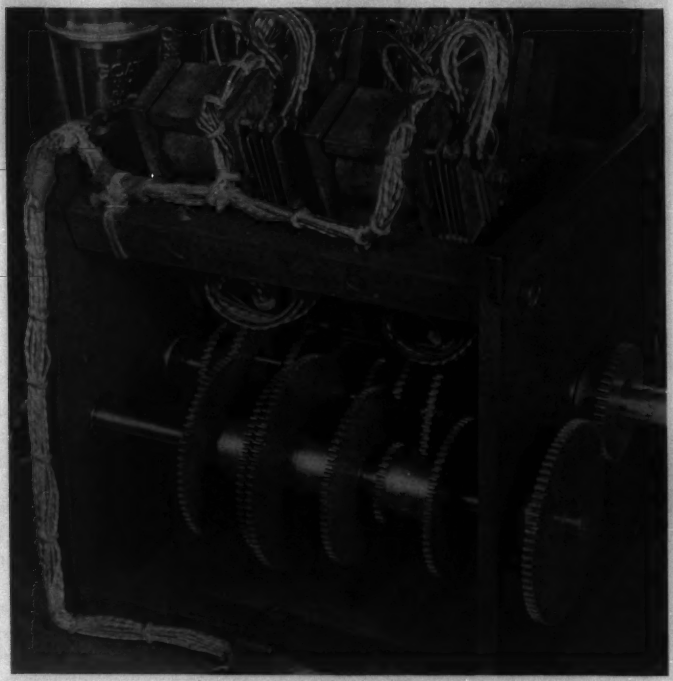


FIG. 25. One-digit decimal gear box.

6.4 One of the most time-consuming operations in setting up the original type of differential analyzer is the insertion of gearing. Both the sizes of gears and their distribution must be planned in advance for every problem and then the individual gears must be placed on

specified shafts. If a problem requires provision for changes of a particular gear ratio, multiple sets of gears are used so that some may be fastened to the shafts and other allowed to idle.

These handicaps are overcome in the new machine by the use of permanent gear clusters, called gear boxes, with electromagnetic shifting devices designed to accomplish changes of ratio, under the control of the B-tape.

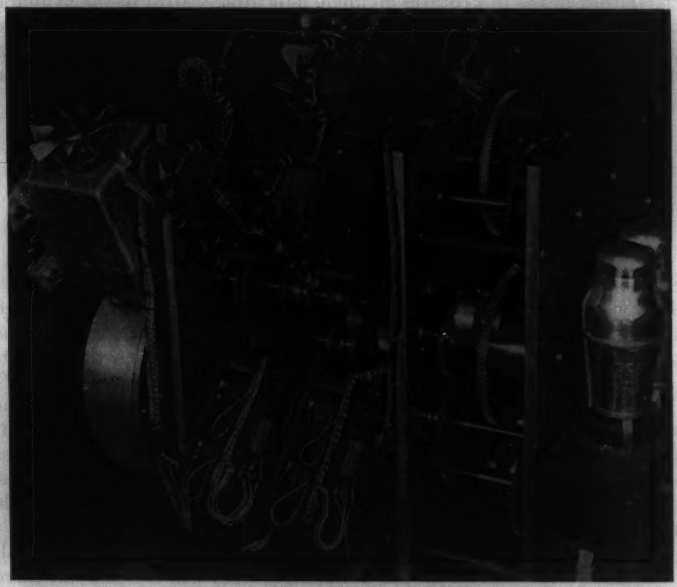


FIG. 26. Decade decimal gear box.

Two types of gear boxes are included in the equipment. A one-digit decimal box, shown in Fig. 25, can be set to any ratio between 0.1 and 1.0, in steps of 0.1. A decade decimal box, shown in Fig. 26, furnishes any ratio between 0.0000 and 1.1110, in steps of 0.0001.

The decade decimal box combines the operation of four one-digit gear sets to form a four-digit decimal number. Only one cluster of input gears is required, and they can mesh with any of the output gears. Each set of output gears furnishes one digit of the specified ratio. The individual digits are then combined by means of differential gears which are designed to form the sum

$$A = B + \tfrac{1}{10}C,$$

where B and C are the input motions and A is the output. Thus, for example, the ratio 0.3715 is established by setting the four sets of one-digit gears to the ratios 0.3, 0.7, 0.1, and 0.5, and then forming, by means of three differential gears, the sum

$$0.3 + \tfrac{1}{10}[0.7 + \tfrac{1}{10}(0.1 + \tfrac{1}{10}0.5)] = 0.3715.$$

Every gear box is driven by one of the standard servomotors, and the output of the box rotates an angle indicator. Most of the available boxes are permanently connected to the inputs of differential gears, so that addition and subtraction require no further gearing. A typical combination of this sort is illustrated by Fig. 27. Two decade gear

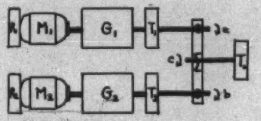


FIG. 27. Gear box and adder combination unit.

boxes, G_1 and G_2, are driven by their servomotors, M_1 and M_2, under the control of angle indicators R_1 and R_2. The variables represented on the shafts of these boxes are "a" and "b" respectively. If the gearing is required merely for use by the separate variables, the outputs are taken off by the direct-connected transmitting angle indicators T_1 and T_2. The permanently-connected differential gear evaluates the sum or difference of "a" and "b," as the rotation "c," and this output may be taken from the angle indicator T_0. Thus, it is merely a matter of selecting angle indicators to obtain either of the separate motions, or their sum or difference. The shaft motion "c" always is available whether it is significant in the problem or not.

Various combinations of gearing are used in order to meet all anticipated needs. Not all the units have adder gears, and some have three component inputs which can be added. The present distribution of gear combinations is given in Appendix V.

6.5 It has been shown that a plentiful supply of integrators meets many of the needs for function generation. But there is also need for function units to supply many types of experimental and empirical functions, and others which are not conveniently generated by integrators and gears. Figure 28 shows the manual function unit used in such cases.

A curve of the required function, plotted in cartesian co-ordinates, is wrapped around a drum. A receiving servomechanism rotates the drum while an operator uses a hand wheel to drive an index "pointer" parallel to the axis of the drum, keeping the index on the plotted curve. The motion of the hand wheel rotates a standard transmitting angle

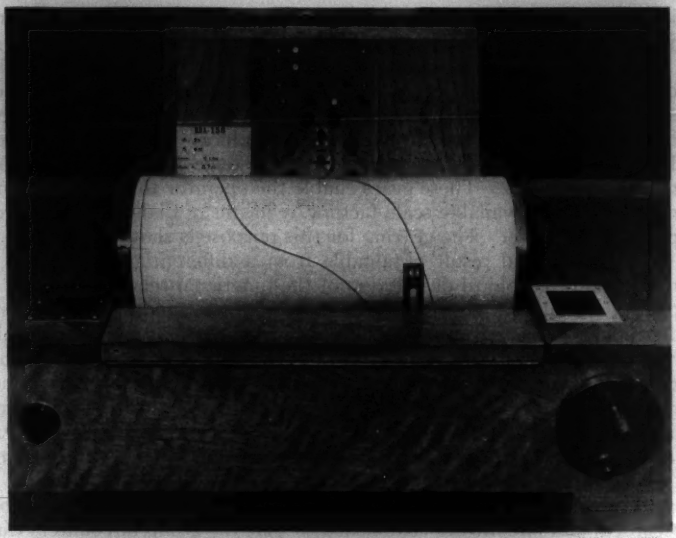


Fig. 28. Manual function unit.

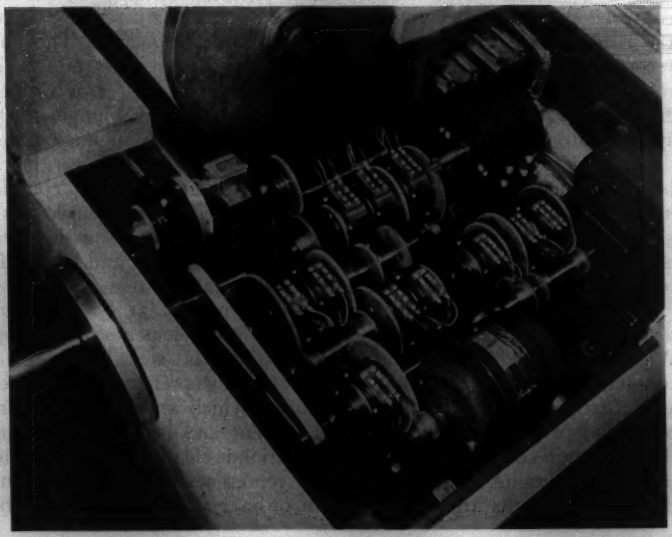


Fig. 29. Servomechanism drive and scale-factor gears for manual function unit.

indicator. A flywheel on the operator's handle is used to prevent jerky motion from entering the servomechanism.

These tables are also arranged so that instead of the normal use as described above, the operator's hand wheel can be driven by another servomotor. By replacing the index with a pen, this table can then be used for plotting an output curve if graphical recording is desired. Both co-ordinate drives are provided with change gearing, operated by magnetic clutches. These gears enable the operator to set up any one of a number of available scale factors by means of dials on the control panel of the table. The gearing for this purpose is shown in Fig. 29.

6.6 The useful result obtainable from a differential analyzer is a numerical or graphical description of the functional relation between a pair of variables. Generally the user wants to know one or more dependent variables as functions of the independent variable. This result is provided by making sets of simultaneous readings of the positions of all the shafts which represent the desired variables, and recording those values. The reading process is practically instantaneous, for it must be accomplished without stopping shaft rotations. An intermediate storage system then holds the readings while a recording typewriter prints them.

A separate counter unit is servo connected to each shaft to be recorded, and circuits are established so that the group of counters can be actuated simultaneously, either in response to a signal from the interval mechanism on the independent-variable drive, or by manual control. After certain "corrections" are applied, as discussed below, the readings are passed into intermediate storage relays. The final recording is done by an electric typewriter which "reads" the storage relays and writes down the result. A typical record sheet of a differential analyzer solution, thus recorded, is shown in Fig. 30. The system automatically picks up the variables in proper columnar order, does all the spacing, and other typewriter operation. Moreover, it will be seen that negative numbers are written as such, and not in complementary form. If any number or digit is missed, the system leaves a gap at the point of failure so that the true location of the missing item is known. Thus, if the number 46285 is to be recorded, and the 6 fails to register, the record is printed 4 285, not as 4285.

The main counter mechanism is shown in Fig. 31. A standard servo unit on the bottom drives a set of contact elements on the top. Whole turns and tenths of turns are detected and stored by a set of contacts which control a group of cold-cathode gas relay tubes. When an actuating signal is received, the tubes which are at that instant connected to the signal line conduct current and because of the gas they maintain conduction until the indication is cleared by interrupting their plate circuits. The digits which represent tens, hundreds, and thousands of turns are obtained from ordinary contacts operated by

067019
067020
074444
098728
098728
054093
054093
074176

M. I. T. DIFFERENTIAL ANALYZER #2
Run No. 68 RDA No. 158 Ed. No. 1
Date 7/26/45 Time 9:18

030297
030295
020123
099994
099994
-039307
-039307
024642

01000	00168	00135	03351	02705	03709	
02000	00491	00390	03128	02422	03130	
03000	00795	00620	02960	02188	02709	
04000	01085	00830	02854	02004	02434	
05000	01367	01022	02774	01842	02219	
06000	01640	01198	02705	01691	02036	
07000	01908	01360	02643	01546	01876	
08000	02169	01507	02587	01407	01735	
09000	02426	01641	02534	01272	01610	
10000	02677	01762	02486	01142	01498	
11000	02923	01869	02441	01015	01399	
12000	03164	01965	02398	00891	01310	
13000	03403	02048	02358	00770	01232	
14000	03637	02118	02320	00651	01163	
15000	03867	02178	02283	00534	01101	
16000	04093	02225	02249	00420	01048	
17000	04317	02261	02215	00307	01002	
18000	04537	02287	02183	00196	00963	
19000	04753	02301	02152	00087	00930	
19800	04924	02305	02128	00001	00907	
20000	04967	02304	02122	-00020	00903	
21000	05178	02297	02093	-00127	00881	
22000	05386	02278	02065	-00231	00865	
23000	05591	02250	02036	-00335	00854	
24000	05793	02211	02009	-00437	00847	
25000	05993	02163	01982	-00537	00845	
26000	06189	02104	01955	-00636	00847	
27000	06384	02036	01928	-00734	00853	
28000	06575	01957	01901	-00830	00863	
29000	06764	01869	01874	-00925	00875	
30000	06950	01772	01847	-01019	00892	
31000	07133	01666	01821	-01110	00911	
32000	07314	01550	01793	-01200	00933	
33000	07492	01425	01766	-01288	00957	
34000	07668	01292	01738	-01374	00984	
35000	07840	01150	01711	-01458	01012	
36000	08010	01000	01682	-01540	01043	
37000	08177	00842	01653	-01620	01074	
38000	08341	00677	01625	-01699	01107	027600
39000	08502	00502	01595	-01774	01141	027598
40000	08660	00322	01566	-01847	01175	021330
41000	08815	00133	01536	-01918	01209	108973
*41684	08919	00000	01514	-01965	01233	108974
42000	08967	-00062	01505	-01986	01244	-044634
43000	09116	-00264	01474	-02051	01279	-044634
44000	09262	-00473	01444	-02114	01314	027563
45000	09405	-00687	01412	-02174	01347	
46001	09544	-00908	01380	-02231	01379	

FIG. 30. Typical record from the new differential analyzer.

cams. These contacts operate storage relays which hold the reading until the recording is completed.

The counting and recording system will handle any variable within a range of ± 10,000 revolutions and record to 0.1 of a revolution. Since the counter deals with a number system, it has a physical zero. Furthermore, when a counter is "picked up" by the control system and servo connected to a variable, it will be in position to indicate, at random, any one of 200,000 possible numbers. It is forced to indicate the correct initial value, and to continue indicating correct values by the following procedure.

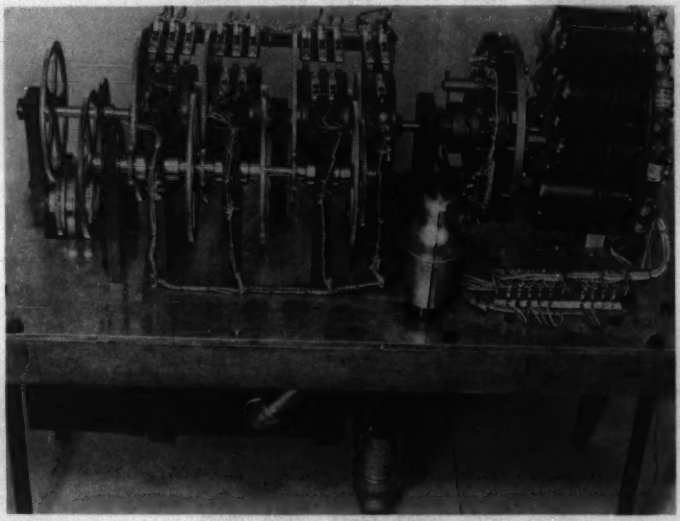


FIG. 31. Counter mechanism.

A relay matrix is provided which can add or subtract any two numbers identified by energizing discrete wires. The result of this operation can be supplied to a set of storage relays.

Initially the counter is indicating some random number I_0. The C-tape says that the initial value of the variable should be R_0. There is therefore a bias number B which when added to the counter indication gives the required initial record. The value of B is found by feeding the counter indication I_0, and the C-tape initial value R_0 to the computing matrix. The matrix carries out the operation

$$B = R_0 - I_0,$$

and stores the result in a set of storage relays associated with the counter. This process is carried out automatically for each counter in use.

Thereafter, during normal operation, each counter indication I_n is fed to the matrix along with the stored correction bias B, and the matrix carries out the operation

$$R_n = I_n + B.$$

The number R_n is the true value of the variable. It is transferred from the matrix to a set of temporary storage relays, where it is held until the correct typewriter solenoids are actuated.

The typewriters used are of a motor-driven type, made by the International Business Machines Corporation and are capable of high-speed operation by remote control. They can be seen in Figs. 41 and 42.

In order to operate the machine in three sections, there are three complete recording channels available, including separate typewriters, computing matrices, decoding relays, and typewriter storage relays. Any channel may use any of the counters and their associated relays. In the main control system, provision is made so that the column in which the record of any variable appears may be specified in advance, thus controlling the order in which counters are read. A set of 10 seven-digit numbers (including spaces and signs) can be recorded every twelve seconds.

For graphical recording there is also another curve-plotting unit available, in addition to those mentioned in section 6.5. The additional unit is built in to the main control table and is designed for servo connection to any two variables which are to be plotted.

7.0 Electrical transmission of shaft position data was specified for the new differential analyzer as the first step in speeding up the process of interconnection. This decision made necessary an electrical switchboard, and at the same time offered the opportunity for the introduction of many desirable automatic features. Some of these have been described in the preceding sections. This section will outline the major features of the main control system, and describe other aspects of the machine as a whole.

7.1 All information, with the exception of that contained in plotted curves, is supplied to the machine by means of punched tapes. A sample of the A-tape, which contains the directions for interconnecting the various computing units, is shown in Fig. 32.

The first entry on the A-tape is the serial number. This number is unique to the particular problem being solved, and the control system uses it in order subsequently to route information to the mathematical units and circuits associated with the problem. The serial number also

appears on the B- and C-tapes, so that gear ratios and integrator settings are likewise routed to the correct units.

After the serial number the A-tape lists by code designations all of the transmitter-receiver connections which are required. Each connection occupies a separate "frame" of the tape. A four-digit code number describes each transmitter or receiver. The first two digits specify what kind of unit is involved and what shaft in that unit; the second two digits specify the serial number of the particular unit of that type, as arbitrarily assigned on a schematic diagram (note the arbitrary numbering of integrators in Fig. 5).

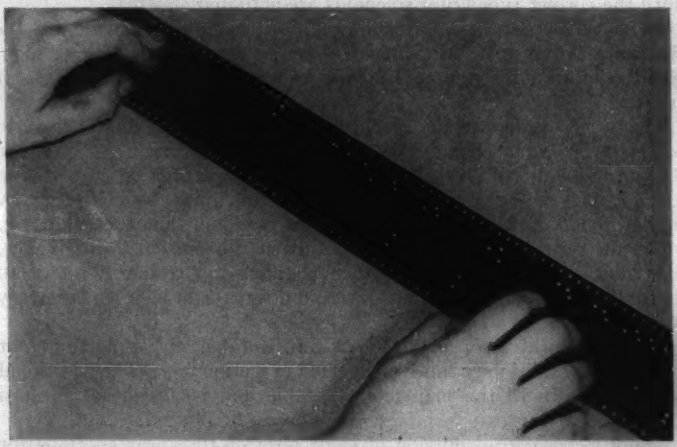


FIG. 32. Section of an A-tape.

For example if the transmitter on a decade decimal gear box (code number 60), which happens to be called the third unit (code 03) of this type on the diagram, is to drive the integrator lead screw (code 80) of the 12th integrator on the diagram, the tape is punched to read 6003–8012 for that frame.

It should be emphasized here that the unit numbers punched on the A-tape are for purposes of identification only. These numbers do not specify the actual units to be selected in the machine. Selection is performed automatically on the basis of the types of units specified by the tape.

When a single transmitter feeds more than one receiver, these connections are grouped in consecutive frames, with the transmitter designated only in the first frame. The relay equipment interprets

this arrangement to mean a multiple connection. The receiver designation also carries an extra hole punched to indicate whether the direction of rotation is to be normal (+) or reversed (−).

Fig. 33. Special tape punch. This machine is used for punching A, B, and C-tapes.

The relay equipment which decodes and executes the tape orders assigns idle units of the type specified, and then "remembers" the serial numbers which were assigned by the tape to each unit selected. Since each problem starts its serial list of unit numbers at one, there will be duplication when the machine is used for more than one problem.

No confusion occurs because the tape first states the serial number of the problem and groups equipment assignments under that number.

When the B-tape is punched with gear ratio data, it also must include the problem serial number and the individual unit numbers. Likewise the C-tape carries first the number of the problem and then the numbers of the individual units to which the initial settings apply. The control system routes all this information automatically to the units assigned by the A-tape.

These various tapes are readily prepared by the special punch shown in Fig. 33. The units in which the tapes are inserted for reading and execution are shown in Fig. 34.

Relays, mostly of the telephone type, have been used profusely in this machine. Figure 35 shows some of the several thousand installed. There are over 1,000 relays in this view, most of them concerned with the duty of receiving tape orders and executing them.

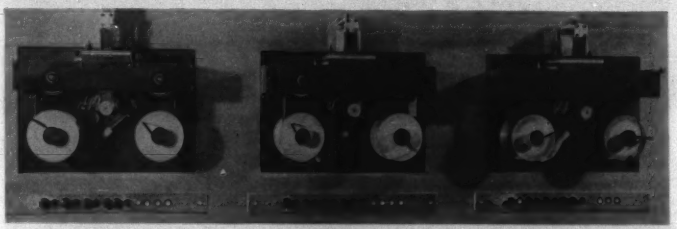


FIG. 34. Tape-reading units for A, B, and C-tapes (left to right).

The actual interconnections are performed on the group of cross-bar switches [14] shown in Fig. 36. A trunking system similar to telephone practice is used in order to provide paths by which any transmitter can reach any receiver. Special design is needed because of the many simultaneous connections required, and also because many of these connections involve one transmitter supplying a number of receivers. It is of interest to note that the first connection to be established, between a transmitter and one receiver, can be completed by any one of 50 different paths.*

In using a system of such scope it is inevitable that operators will commit errors. Perhaps the outstanding feature of the control-system design is the use of interlocks which automatically bar improper operation which might cause damage or error.

[14] Scudder, F. J., and Reynolds, J. N., "Cross-Bar Dial Telephone Switching System," *Trans. A. I. E. E.*, 58: 179–192, May 1939.

* The Bell Telephone Laboratories gave to the project most of the experimental model of their first cross-bar machine switching system after they had completed studies on it. Most of the main control system was built with parts from this much appreciated gift.

Fig. 35. Some of the relays in the main control system.

Fig. 36. A group of cross-bar switches in the main control system.

After a problem has been set up on the machine, the assembly is operated at the main control station. This consists of the three identical panels shown on Fig. 37. Each panel has supervision over a single assembly constituting one problem.

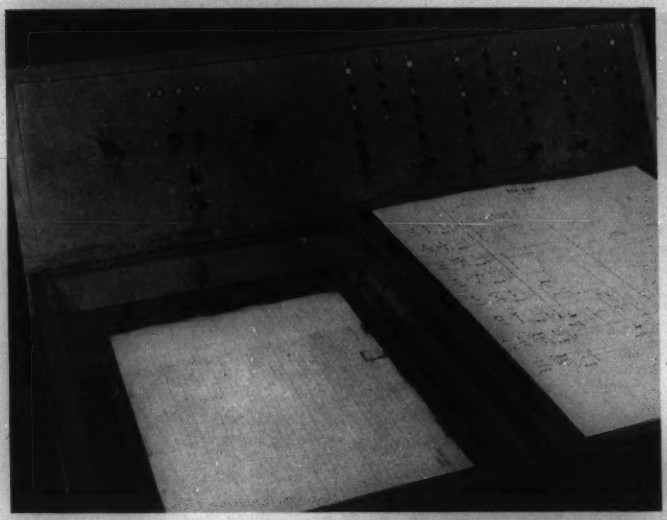


FIG. 37. Main control station.

There is no restriction on the manner in which the equipment may be divided into sections for work on more than one problem. If a new problem is ready for the machine while another problem is already on, the new tapes can be inserted, and the control system will do the assembly, regardless of the distribution of equipment, provided there are enough idle units available of the types required.

Where gear boxes are required a special situation exists. For example a one-digit box may be called for, but none are available. However, a decade box is available. In such cases, and similar ones involving gear and differential combinations, the control equipment always seeks more versatile units which will satisfy the technical requirements in any case when the less versatile units are not available.

7.2 In the design of a machine of this type the means for locating and diagnosing malfunctions must be provided early and made a part of the assembly. The main power supply equipment is provided with conventional protective devices. At distribution panels, however,

power is supplied to separate circuits through alarm-type fuses which are designed for quick location of failures.

Servomechanism panels are supplied with jacks connected to all critical circuit points. Test equipment mounted on a rolling table can be plugged in for examination, adjustment and repair. Similar testing plugs and portable test equipment are provided for automatic units such as setting mechanisms and gear boxes.

When trouble occurs in the solution of a problem it is sometimes necessary to know what particular mechanical units are involved. A supervisory control panel, shown in Fig. 38, enables the operator, by merely moving keys and pushing buttons, to determine the role assigned to any piece of equipment in the machine, or by another manipulation to find which piece of equipment represents any part of the problem.

All units are provided with "busy" keys which are thrown when a unit is to be tested or removed for shop work. This prevents the control system from attempting to use that piece of equipment. No interference with operation results because the A-tape will always select available units of the types required. Signal lamps are installed at all units to indicate whether they are in use. Other lamps indicate what stage in a sequence of operation has been reached, and still others indicate various kinds of possible faulty operation.

A servomechanism failure must be detected immediately because it invalidates the solution at the point where it occurs. A special circuit is included in each receiver panel to detect loss of synchronism. This equipment sounds an alarm, lights a lamp to show where failure occurred, and shuts down the independent variable drive.

Using electrical servo connections entirely, the mechanical computing units may be located wherever convenient. In Fig. 39 integrators and their setting mechanisms are in the bottoms of the equipment bays. A complete unit may be withdrawn onto a "dolly" and removed. The upright members of the bays form racks which support setting mechanism and gear-ratio relays on one side, and transmitter and receiver panels on the other. Figure 40 shows one group of servomechanism panels.

Gear box combinations may be seen in Fig. 39 located above the integrators. Overhead tracks are provided so that these units can be rolled out for maintenance. Function units and counters are likewise housed in bays with electrical equipment mounted on the vertical side members.

7.3 Figure 41 is a view of the new differential analyzer as seen from the operating area. In the foreground are the manual function units and just beyond them are: on the left, the punched tape mechanisms; and on the right, the main operating table with its control board and the recording typewriters. In the background are the equipment bays

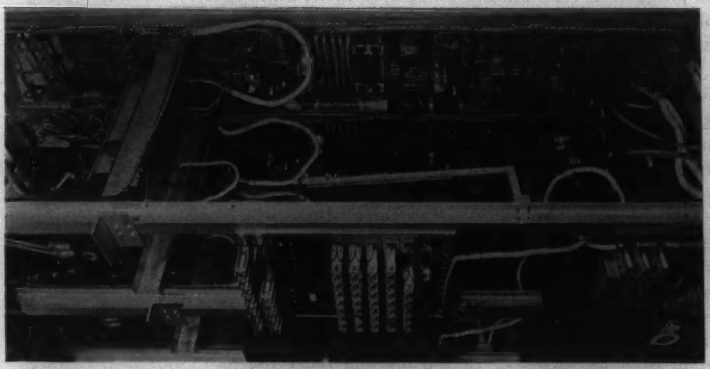


FIG. 39. Typical equipment bay. Gear boxes above and integrator below.

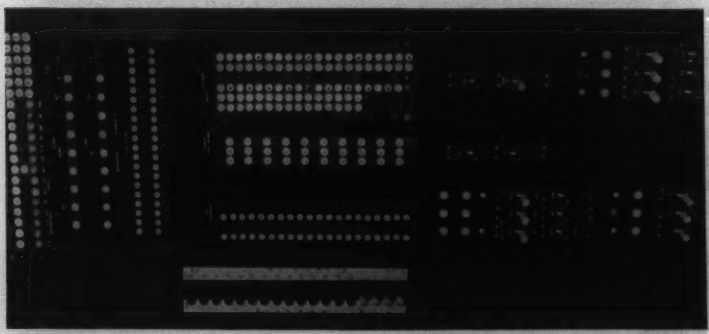


FIG. 38. Supervisory control panel.

Fig. 40. One group of servomechanism control panels.

FIG. 41. General view of machine.

which contain the integrators and gear boxes. In Fig. 42 the control relays, cross-bar switches, and recorders are on the right.

This machine is now in useful operation, but it is not "completed," and it will be incomplete for many years. During the years of war use, it has not been possible to carry out many investigations regarding its ultimate performance. Yet, enough experience has been gained to indicate that the machine delivers its projected performance of precision to one part in 10,000, at least in the sense that for those cases where such precision has not been reached, there are others where it has been comfortably exceeded. In general the precision available in the new machine is adequate, and it can be increased in some cases if necessary.

This whole matter of machine precision in the solution of differential equations is an exceedingly difficult subject to discuss with assurance. Much, if not most, of the behavior of the mechanism depends upon the nature of the equation. Certain equations give trouble even in numerical integration, and computations have to be carried out using superfluous significant figures merely to avoid accumulation of error. In other words some problems have inherent "stability" and others do not. The latter class is difficult to handle by any method. Solutions of "stable" types of equations have been obtained on the new machine to a probable error of one part in 25,000.

The prospects for future improvement are reassuring. Useful results are being obtained from about 100 tons of equipment, containing about 2,000 electronic tubes, several thousand relays with an untold number of contacts, about 200 miles of wire and about 150 motors. Despite its complex construction the machine operates reliably over long periods. Less than five per cent. of its useful time is lost due to malfunctions, and most of this loss can be saved by changes now under way.

The advantages of the new machine for handling short problems have clearly been demonstrated. In one actual case a fresh problem was brought to the machine in the morning and eight solutions were furnished during the day. Within this time all preparatory work was done, including preparation of the assembly diagram and connection sheets, calculation of gear ratios and scale factors, and tape punching.

Even for large assemblies a complete set of tapes can be punched in less than fifteen minutes. Subsequent solutions usually require only a new C-tape (or part of a C-tape). For average problems the machine is ready to run within about fifteen minutes after the tapes are available.

8.0 A new type of differential analyzer has been described which by its enlarged capacity, its improved precision, and especially its flexibility and adaptability, offers new opportunity and new challenge to the work of the engineer, the mathematician, the theoretical and the experimental scientist.

Much has already been accomplished by these groups using the

FIG. 42. General view of machine.

earlier type of machine for the implementation of mathematical analysis. But there remains an unexplored area of thought and investigation in which the machine becomes a part of mathematical expression. The great variety of form and structure possible even in simple differential equations offers the means for creating and using entirely new types of functions, defined only by their differential equations. Much of analysis based on linearity is being extended and requires further extension to the non-linear realm.*

Certainly, with this automatic device for solving his differential equations, the mathematical worker can leave behind some of the inhibitions of classical analysis. To be sure, he gets from the machine by present methods only a numerical result, but that is no small reward. On the frontiers of research science a sound number is indeed a shining goal.

9.0 This machine was created by the efforts of an able and loyal staff. In the design of the main control system and in shaping major features of the general structure and operation, Mr. Richard Taylor contributed distinguished work. As executive officer of the Center of Analysis of the Massachusetts Institute of Technology since 1940, he has been responsible for the management and modification of the machine in its initial use.

Mechanical design and most of the electro-mechanical designs were the work of Mr. L. E. Frost and Mr. H. R. Morash. Mr. Frost was in charge of all mechanical shop work and carried the principal responsibility for precision execution, with the able assistance of Mr. Maurice Forbes, expert machinist.

Mr. J. J. Jaeger designed a considerable part of the electronic equipment and was in charge of electrical construction and assembly.

One of the trying tasks in this enterprise was that of locating and eliminating the many errors which are inevitable in such complex construction. The success of this phase was largely due to the skilled perserverance of Messrs. J. L. C. Löf and F. M. Verzuh, with the assistance of J. E. Goos.

This machine was built largely by means of a grant from the Rockefeller Foundation. The project has enjoyed the continuous support and encouragement of the Foundation's officers, and in particular that of Dr. Warren Weaver, Director for the Natural Sciences.

APPENDIX I.

Generation of Functions.

Section 5.3 presents a basic method for generating a function by defining it as the solution of an ordinary differential equation. The

* For example, a differential analyzer is particularly efficient for studying the effects of nonlinearity in mechanical and electrical networks. Straightforward methods are applicable in dealing with dry friction, backlash, hysteresis, and discontinuities produced by the action of switches, valves and limit stops.

additional examples given below include the commonly used algebraic, trigonometric, exponential, and logarithmic functions.

The method given for generating x^2 may be extended to higher powers, and to polynomial forms. Frequently, more than one procedure is possible for generating a polynomial. In such cases the preferred procedure is usually one which uses a minimum of equipment, especially of integrators.

For example, the following methods might be used to generate the function $y = x^4 + 2x^2$:

(a) Evaluate $\int x dx$ to get x^2, then $\int x^2 d(x^2)$ to get x^4. Add x^4 and $2x^2$. (Two integrators, one adder.)

(b) Solve $\frac{d^4y}{dx^4} = 24$. (Four integrators.)

(c) Solve $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 12y - 4 = 0$. (Three integrators, one adder.)

(d) Solve $\frac{dy}{d(x^2)} = 2x^2 + 2$, after obtaining x^2 from $\int x dx$. (Two integrators.)

Methods (a) and (d) are very nearly the same, but the elimination of the adding gear in method (d) is desirable.

Negative powers may be produced by first obtaining the reciprocal. Let $y = 1/x$, then $dy/dx = -y^2$ is the defining equation. The connections given in Fig. 43 generate both x^{-1} and x^{-2}.

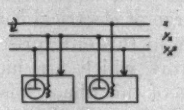


FIG. 43. Generation of the reciprocal.

A more general treatment of powers is accomplished by way of the logarithm.

Let

$$y = \ln x,$$

then

$$\frac{dy}{dx} = \frac{1}{x},$$

and

$$\frac{d^2y}{dx^2} = -\frac{1}{x^2} = -\left(\frac{dy}{dx}\right)^2.$$

The defining equation may now be written

$$\frac{dy}{dx} = -\int \frac{dy}{dx} \cdot \frac{dy}{dx} \cdot dx = -\int \frac{dy}{dx}\, dy.$$

Figure 44 shows how these relations are used to generate both the reciprocal and the logarithm.

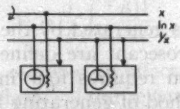


FIG. 44. Generation of logarithm and reciprocal.

After generating $\ln x$ an ordinary gear ratio can be used to produce $n \ln x$. This can then be used in the equation

$$x^n = \int x^n d(n \ln x),$$

as shown in Fig. 45. Thus any real power of a variable, positive or negative, fractional or integral, may be generated.

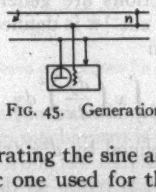


FIG. 45. Generation of x^n.

Connections for generating the sine and cosine are shown in Fig. 10. This assembly is a basic one used for the production of all sinusoidal functions. The only changes are those required to introduce different coefficients or additive constants. For example, in order to produce

$$y = \sin^2 x,$$

take

$$\frac{dy}{dx} = 2 \sin x \cos x,$$

and

$$\frac{d^2y}{dx^2} = 2(-\sin^2 x + \cos^2 x) = -2(2 \sin^2 x - 1),$$

or

$$\frac{d^2y}{dx^2} = -4y + 2.$$

This equation differs from the equation of Fig. 10, $d^2y/dx^2 = -y$, only in the coefficient of y and the additive constant; hence the same basic connections are used.

The tangent is defined by the equation

$$\frac{dy}{dx} = y^2 + 1,$$

and the cotangent by

$$-\frac{dy}{dx} = y^2 + 1.$$

Either of these functions is generated by the connections of Fig. 46.

Both the secant and cosecant are defined by the equation $d^2y/dx^2 = 2y^3 - y$. This equation requires four integrators for its solution. Hence the alternative method of generating from $\sec x = 1/(\cos x)$ and $\csc x = 1/(\sin x)$ is just as economical of equipment. It is the preferred method where both the function and its reciprocal are required.

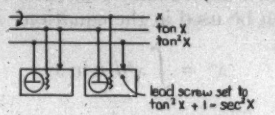


FIG. 46. Generation of tan x.

Inverse trigonometric functions are generated by straightforward procedure. For example, $y = \sin^{-1} x$ is defined by $d^2y/dx^2 = x(dy/dx)^3$, or in the more convenient form

$$\frac{dy}{dx} = \int x \left(\frac{dy}{dx}\right)^2 dy.$$

Likewise, $y = \tan^{-1} x$ is defined by $d^2y/dx^2 = -2x(dy/dx)^2$, and is generated from the equation

$$\frac{dy}{dx} = -2 \int x \frac{dy}{dx} dy.$$

Compound functions should be treated with caution. For example, $y = \epsilon^x \cos x$ could be generated by forming ϵ^x and $\cos x$ separately, and multiplying, a process which requires four integrators and one adder.* But the function is also defined by $d^2y/dx^2 = 2[(dy/dx) - y]$, and this equation can be solved by using two integrators and one adder.

APPENDIX II.

Inverse Integration.

In section 3.5 the integrator was shown to operate in accordance with the equation

$$\frac{dW}{dV} = kU.$$

* Note that $\int \epsilon^x d(\cos x) + \int \cos x d(\epsilon^x) = \int (\cos x - \sin x) d(\epsilon^x)$.

Normally, this is stated in the form

$$W = k\int_{V_1}^{V} U dV.$$

Another mode of operation, derived from the same equation, is inverse integration, for which the equation is stated

$$V = \frac{1}{k}\int_{W_1}^{W} \frac{dW}{U}.$$

This type of operation requires, in effect, that the disc of the integrator be driven by the wheel. True reversibility of this short is not possible with the integrator as actually built, but the equivalent of reversible operation is obtained by special connections of the components of the associated servomechanism equipment.

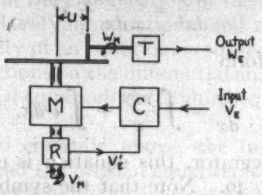


FIG. 47. Servo connections for normal integration.

Figure 47 shows the situation when the integrator is used normally. Since the lead screw servo mechanism is not changed in going to inverse operation, it is omitted from the diagram, and the displacement U is merely indicated. It will be noted that the electrical input signal V_E enters the control panel C where it combines with the signal V'_E from the angle indicator R and thus causes the motor M to furnish the mechanical rotation V_M. The angle indicator T transforms the mechanical rotation W_M of the output wheel into the electrical signal W_E.

In Fig. 48 the servo connections for inverse operation are shown. Now, the input is the electrical signal W_E. This is combined in the

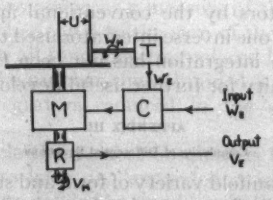


FIG. 48. Servo connections for inverse integration.

control circuit C with the signal W'_E, produced by the angle indicator on the output wheel shaft. The servo motor M must now furnish a mechanical rotation V_M which causes the output rotation W_M to satisfy the input W_E. Hence the effect is the same as though a motion W_M applied to the wheel were driving the disc. The mechanism is thus connected to operate in accordance with the equation for inverse integration. The output V_E is obtained from the angle indicator R.

In the design of the main control system of the new machine provision is made for inverse operation of integrators by properly punching the A-tape.

Inverse integration provides a means for simplifying the connections for many problems which involve division of variables. Instead of forming a reciprocal and multiplying, the equation can often be written so that inverse integration can be used. For example, the equation

$$\frac{d^2y}{dx^2} = \frac{y}{x}\frac{dy}{dx} + \epsilon^v$$

can be written in the form

$$\frac{dy}{dx} = \int \frac{y dy}{x} + \int \epsilon^v dx.$$

Using one inverse integrator, this equation is evaluated by the connections shown in Fig. 49. Note that the symbol used for an inverse integrator indicates that the middle shaft is now the source of torque.

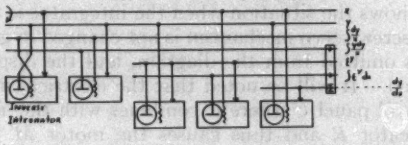


FIG. 49. Application of inverse integration to $\frac{d^2y}{dx^2} = \frac{y}{x}\frac{dy}{dx} + \epsilon^v$.

Further application of inverse integration appears likely in the generation of functions. For example, the function $y = \ln x$ which required two integrators by the conventional approach (see Fig. 44) can be generated with one inverse integrator used to produce $y = \int dx/x$. This phase of inverse integration has not been fully exploited and no doubt offers opportunity for further useful development.

APPENDIX III.

Complexity of Differential Equations.

Considering the manifold variety of form and structure in differential equations, the "order" of an equation is an inadequate description at

best. It is a classification scheme based on the analytical attack. Using the machine approach, a different sort of classification is somewhat more revealing.

The inadequacy of order as a description lies in the fact that it describes one term of an equation and ignores all others. The equations

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$$

and

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + b \sin y = 0$$

are both of second order, and in structure they are very similar, yet one yields an analytic solution readily and the other does not. Evidently the change from "y" to "$\sin y$" is a profound one and should, if possible, be reflected in the classification of the equations.

A new method of classifying equations, and one which contains a measure of the complexity of an equation, is suggested by the procedure used for generating functions on the differential analyzer. If a function is defined by a differential equation, the defining equation can be written instead of the function.

Thus, in the second equation above, the function $\sin y$ may be replaced by $z = \sin y$, and the original equation is then replaced by the following set of equations:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + bz = 0$$

and

$$\frac{d^2z}{dy^2} + z = 0.$$

By this step the original second-order equation is expanded into two second-order equations. The objective in the expansion process is to obtain a set of equations each of which contains only linear terms. When this objective has been reached the order of each component equation becomes significant.

It seems reasonable to assign as a measure of the complexity of the parent equation, a number equal to the sum of the orders of the set of equations obtained after all functions are replaced by their lowest-order defining equations. The number thus obtained is in most cases equal to the number of integrators required in the differential analyzer assembly.

In the above example the number obtained is four, and the parent equation may be said to have an "expanded order" or four. It may perhaps be more useful to retain the word "order" as a general descrip-

tion of the parent equation, and supplement it by giving the value of the "function order" separately. Thus the above case is one or a second order equation with second order functions.

Consider as another example the equation

$$\frac{d^2y}{dx^2} + e^x \frac{dy}{dx} + y^2 \sin x = 0.$$

This equation expands into the set of equations:

$$\frac{d^2y}{dx^2} + \alpha \frac{dy}{dx} + \beta\gamma = 0,$$

$$\frac{d\alpha}{dx} - \alpha = 0,$$

$$\frac{d\beta}{dy} - 2y = 0,$$

and

$$\frac{d^2\gamma}{dx^2} + \gamma = 0.$$

In accordance with the ideas expressed above the parent equation of this set may be described as having an expanded order of six, or as a second order equation with fourth order functions.

As another example the equation given in section 3.1,

$$\frac{d^3y}{dx^3} = \left(2y \frac{d^2y}{dx^2} - \sin \frac{dy}{dx}\right)^2,$$

can be expanded into the set of equations

$$\frac{d^3y}{dx^3} = Q,$$

$$\frac{dQ}{d\eta} = 2\eta,$$

$$\eta = 2y \frac{d^2y}{dx^2} - z,$$

and

$$\frac{d^2z}{d\left(\frac{dy}{dx}\right)^2} = -z.$$

In this case the expanded order is eight, or the parent equation may be described as being of third order with fifth order functions.

APPENDIX IV.

Scale Factors.

In setting up a differential analyzer assembly gears must be chosen so that the displacements of integrators and function units are kept within design limits. Moreover, it is essential to precision that the number of revolutions made by each shaft in representing its variable be adequately large. Finally, the time of solution should be as low as possible, without sacrificing precision. These requirements are taken into account in determining scale factors.

The calculation of scale factors is largely a matter of arithmetic. A few basic principles must be observed, and these will be illustrated by an example.

The scale factor of any shaft is defined as the number of revolutions turned by that shaft per unit of the variable represented. If a shaft represents a variable y at a scale factor of 1,500, and y varies by 3.2 units, the shaft will make $3.2 \times 1{,}500 = 4{,}800$ revolutions.

In schematic diagrams each shaft is labelled by the variable it represents. When scale factors are known, each shaft is labelled by the product of the scale factor and the variable. In the example above the shaft would be labelled $1{,}500y$.

A gear ratio operates directly upon the scale factor. Thus, if a $1{,}500y$ shaft is connected to a gear box set for a ratio 0.62, the output of the gear box is $0.62 \times 1{,}500y = 930y$.

A constant coefficient can be introduced either by means of a gear ratio, as above, or by change of scale factor. Thus, if a $1{,}500y$ shaft is available, and the problem specifies the need for $y/4$, where $1/4$ is a coefficient, a gear box may be set for the ratio 0.25 and used to produce

$$\frac{1{,}500}{4}\, y = 1{,}500 \left(\frac{y}{4}\right).$$

In many cases, however, it is unnecessary to use an actual gear, for a shaft labelled $1{,}500y$ can also be used as

$$1{,}500 \times 4 \left(\frac{y}{4}\right) = 6{,}000 \left(\frac{y}{4}\right).$$

Variables which are to be added or subtracted must enter the differential gearing at the same scale factor. Hence gearing must be provided in association with all adder gears (see Appendix V).

The scale factor of any variable entering the lead screw of an integrator must be such that the extreme excursion of the variable from zero is represented by a shaft rotation equal to or less than $\pm$ 150 turns.

The constant "k" of an integrator is 0.01. Thus if the variables U and V are represented to the scale factors A and B respectively, the

integrator equation becomes

$$SW = \frac{1}{100}\int AUd(BV)$$

$$= \frac{AB}{100}\int UdV.$$

Hence, the scale factor of W is given by

$$S = \frac{AB}{100}.$$

In order to calculate scale factors, certain numerical facts must be known about the variables in the problem. The excursion of the independent variable must be given approximately, and the extreme departures from zero must be given for variables appearing on the lead screws of integrators. These numbers need not be given accurately, for exploratory runs can be made using highly conservative scale factors in order to locate extreme values.

As a specific example consider the equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \sin y = 0,$$

for which schematic connections are shown in Fig. 50.

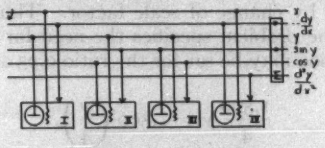


FIG. 50. Connections for $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \sin y = 0$.

Using Fig. 50 as a guide a diagram of the type shown in Fig. 51 is prepared to indicate the locations of the gears required, and the relations among the scale factors.

Since the scale factor of x is unknown, the letter A is assigned to represent it, and the shaft hence carries the label Ax. Likewise the shaft on which dy/dx is produced is assigned the scale factor B. Since this shaft drives the lead screw of integrator I, a gear ratio n_1 is assigned to accommodate the total rotation $n_1B(dy/dx)$ to the limited capacity of the screw ($\pm$ 150 turns). But dy/dx must also drive one input of an adder gear. This requires a gear ratio which is not necessarily the same as n_1, so a new ratio n_2 is assigned. Since the equation requires the term $2(dy/dx)$, the scale factor at the input to the adder is modified

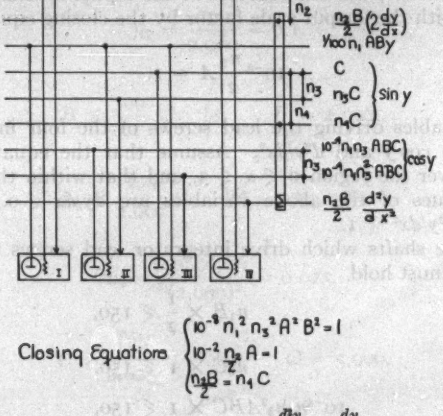
further because

$$n_2 B \frac{dy}{dx} = \frac{n_2 B}{2}\left(2\frac{dy}{dx}\right).$$

The output of integrator I is y, and its scale factor is expressed in terms of the input scale factors already assigned, from the relation

$$\frac{1}{100}\int\left(n_1 B \frac{dy}{dx}\right) d(Ax) = \frac{1}{100} n_1 ABy.$$

Next the shaft for $\sin y$ is encountered, and since its scale factor is unknown, it is labelled $C \sin y$. The gear ratios n_3 and n_4 are assigned for the same reasons as n_1 and n_2 above.



Closing Equations

$$\begin{cases} 10^{-8} n_1^2 n_3^2 A^2 B^2 = 1 \\ 10^{-2} \frac{n_3}{2} A = 1 \\ \frac{n_2 B}{2} = n_4 C \end{cases}$$

FIG. 51. Gearing and scale factors for $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \sin y = 0$.

The scale factor for the output of integrator II is now determined from its inputs, as follows:

$$\frac{1}{100}\int (n_3 C \sin y) d\left(\frac{1}{100} n_1 ABy\right) = 10^{-4} n_1 n_3 ABC \cos y.$$

Since the variation of $\cos y$ is the same as that of $\sin y$, the gear ratio n_3 is used between the output of integrator II and the lead screw of integrator III.

The scale factor at the output of the adder gear must correspond to the input scale factors. This condition is satisfied by assigning the

scale factor $n_2B/2$ to the output shaft and writing the equation $n_2B/2 = n_4C$.

At this point the scale factors assigned to the outputs of integrators III and IV must be made consistent with the values developed from their respective input scale factors.

For integrator III the output scale factor computed from input values is given by

$$\frac{1}{100}\int (10^{-4}n_1n_3^2ABC\cos y)d\left(\frac{1}{100}n_1ABy\right) = 10^{-8}n_1^2n_3^2A^2B^2C\sin y;$$

whereas, the assigned scale factor for $\sin y$ is C. It is necessary to make these values agree by writing the closing equation

$$10^{-8}n_1^2n_3^2A^2B^2 = 1.$$

In similar manner, the scale factors at the inputs of integrator IV are reconciled with the output scale factor by the closing equation

$$10^{-2}\frac{n_2}{2}A = 1.$$

The variables driving the lead screws of the four integrators are dy/dx, $\sin y$, $\cos y$ and d^2y/dx^2. Assume that the equation is to be evaluated over the region $0 \leqslant x \leqslant 3$, and that within this region the extreme values of the above variables are $dy/dx \leqslant 0.5$; $\sin y \leqslant 1$; $\cos y \leqslant 1$; $d^2y/dx^2 \leqslant 1$.

For those shafts which drive integrator lead screws the following inequalities must hold

$$n_1B \times \frac{1}{2} \leqslant 150, \qquad (a)$$

$$n_3C \times 1 \leqslant 150, \qquad (b)$$

$$10^{-4}n_1n_3^2ABC \times 1 \leqslant 150, \qquad (c)$$

$$\frac{n_2}{2}B \times 1 \leqslant 150. \qquad (d)$$

In addition the scale factors chosen must satisfy the three closing equations

$$\frac{n_2B}{2} = n_4C, \qquad (e)$$

$$10^{-4}n_1n_3AB = 1, \qquad (f)$$

and

$$10^{-2}\frac{n_2}{2}A = 1. \qquad (g)$$

It may be noted in the above that if eq. (*f*) is substituted in inequality (*c*), it reduces to inequality (*b*). Hence there are only six relations available among seven unknowns, and there is no closed solution.

A set of values may be found by starting with some reasonable estimate of the time of solution expected and an assumed average running speed of the independent variable.

In covering the specified range of x from $x = 0$ to $x = 3$, the Ax-shaft will make $3A$ revolutions. Assume that a single solution requires five minutes, with the x-shaft operating at an average speed of 1,200 r.p.m. Then

$$3A = 5 \times 1{,}200$$

and

$$A = 2{,}000.$$

From eq. (*g*)

$$n_2 = \frac{200}{2{,}000} = 0.1.$$

From (*d*)

$$B \leqslant \frac{300}{0.1}; \qquad \text{let} \qquad B = 2{,}000.$$

From (*a*)

$$n_1 \leqslant \frac{300}{2{,}000}; \qquad \text{let} \qquad n_1 = 0.1.$$

From eq. (*f*)

$$n_3 = \frac{10^4 \times 10}{(2{,}000)^2} = 0.025.$$

From (*b*)

$$C \leqslant \frac{150}{0.025}; \qquad \text{let} \qquad C = 5{,}000.$$

From eq. (*e*)

$$n_4 = \frac{0.1 \times 2{,}000}{2 \times 5{,}000} = 0.02.$$

Summary of Scale Factors.

$$A = 2{,}000; \qquad B = 2{,}000; \qquad C = 5{,}000;$$

$$n_1 = 0.1; \qquad n_2 = 0.1; \qquad n_3 = 0.025; \qquad n_4 = 0.02.$$

Using these values, the final diagram and connection sheets may now be constructed as a basis for punching the control tapes. This aspect of the subject is described in a manual of operation.

APPENDIX V.

Table of Equipment.

The new differential analyzer contains the following equipment:

Independent variable units 3
Integrators and setting mechanisms 18
High-speed numerical counters 10
Numerical recording channels 3
(including special typewriters)
Manual function units 3
(These units can also be used for graphical recording)
Graphical recording unit 1
Automatic function units (not yet in operation) 4
Gear combinations—all gear boxes can be used separately or with associated adder gears.

The details of gearing available are given in the following table.

Gear Table.

Type of unit and gear boxes contained	Number of units	Total number of decade boxes	Total number of one-digit boxes	Total number of adders
3 input adder unit, with 2 decade boxes and 1 direct drive	2	4	0	4
2 input adder unit, with 1 decade box and 1 one-digit box	6	6	6	6
2 input adder unit, with 1 decade box and 1 direct drive	3	3	0	3
2 input adder unit, with 1 one-digit box and 1 direct drive	3	0	3	3
Non-adding unit, with 1 decade box and 1 one-digit box	3	3	3	0
Grand Totals	17	16	12	16

One-digit boxes provide all ratios from 0.1 to 1.0, in steps of 0.1. Decade boxes provide all ratios from 0 to 1.1110, in steps of 0.0001.

THE CHARGE EFFECT IN RELATION TO THE KINETICS OF PHOTOGRAPHIC DEVELOPMENT. IV. THE VARIATION OF EMULSION SPEED WITH CHARGE. THE ACTION OF COMPLEX DEVELOPERS.

BY

T. H. JAMES.

Communication No. 1037 from the Kodak Research Laboratories.

In a previous paper,[1] it was shown that several developing agents of diverse types are capable of yielding practically the same emulsion speed when development of motion-picture positive film is carried to completion in the absence of sulfite. The agents tested included the uncharged diaminodurene, the singly charged methyl-*p*-aminophenol (elon), catechol, and *p*-aminophenol, and the doubly charged hydroquinone, elon monosulfonic acid, and the ferro-oxalate ion. At intermediate stages of development, however, the shape and position of the characteristic curves obtained with these agents may vary widely. This variation persists when sulfite is added to the developer solution, although it is decreased somewhat in magnitude.

The shape and position of the photographic characteristic curve obtained by incomplete development are determined by the nature of the function which relates the rate of density formation to the exposure. Several factors may play a part in determining this function. We do not know the complete story. However, the charge effect appears to be one of the more important factors. In many cases, the influence of charge can be recognized in the behavior of developing solutions of conventional composition, but for the purposes of correlation, the use of solutions of somewhat simpler composition is preferable. The influence of charge is then more readily isolated.

The type of variation obtained by a change in the charge of the developing agent alone is illustrated in Figure 1. The curves represent data obtained with three chemically similar developing agents: sym-dimethyl-*p*-phenylenediamine, methyl-*p*-aminophenol, and methyl-*p*-aminophenol monosulfonate. The photographic material used was a normal motion-picture positive film (C). The development procedure followed was the same as that described previously,[1] and the solution had the composition: developing agent, 0.001M; KOH, 0.05M; KBr, 0.00667M. Developing times were adjusted to give approximately the same density for the first exposure step ($\log E = 2.05$). On this basis of comparison, the densities obtained at the lower exposures increased markedly in going from the doubly charged agent to the

[1] T. H. James, *J. Franklin Inst.*, 239, 41 (1945).

singly charged one, and a further but small increase occurred in going to the uncharged agent.

The variation in speed may be evaluated in other ways. For example, a definite increase in inertia speed with decrease in charge was noted when comparison was made between curves of equal (low) gamma values. Thus, development to a gamma of 0.65 gave inertia speeds of 2, 32, and 65 for the doubly, singly, and uncharged agents, respectively.

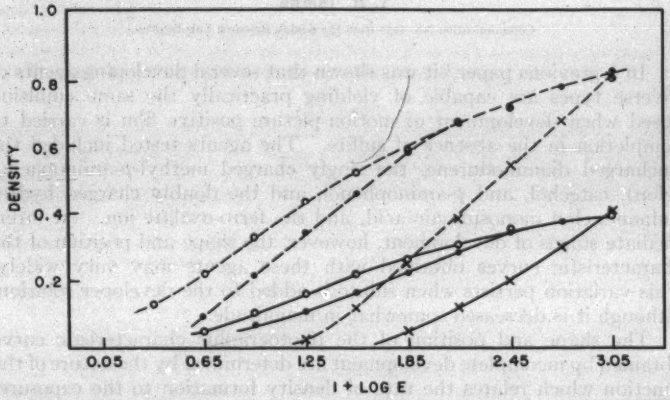


FIG. 1. Effet of charge on the shape and position of the characteristic curve. –○–○– = s-dimethyl-*p*-phenylenediamine; –●–●– = methyl-*p*-aminophenol; –×–×– = methyl-*p*-aminophenol monosulfonate. The two sets of curves refer to two different sets of development times. Composition of solution: developing agent, 0.001 *M*; KOH, 0.05 *M*; KBr, 0.0067 *M*.

A more general comparison of a number of diverse developing agents was made in the presence of 17 grams of sulfite per liter. The sulfite was added to suppress the effect of oxidation products on the development rate. Apart from its action in removing the oxidation product, this amount of sulfite had a slight effect upon the emulsion speed when development was carried to only a low gamma (see subsequent discussion), but the complication arising from this effect was considered to be less disturbing than the complications which arise if the oxidation products are not removed. All agents were employed in the caustic formula already given.

Figure 2 gives a comparison of the characteristic curves obtained with the various agents when development was carried to a gamma of 0.4. The straight-line region of the curves was well defined for the singly and doubly charged agents employed, and the curves for these agents fell into distinctly separate groups. The curves for the singly charged agents fell within the confines of the zone marked "A" in the

figure. Curves for the doubly charged agents lay in zone "B." Methyl-*p*-aminophenol sulfonate, however, gave a substantially greater emulsion speed than any other member of the doubly charged group, but the speed was still considerably less than that for the singly charged agents.

All of the agents for which data are included in Figure 2 yield substantially equal gamma values at maximum development,[1] and most of them yield approximately the same emulsion speed. Two agents

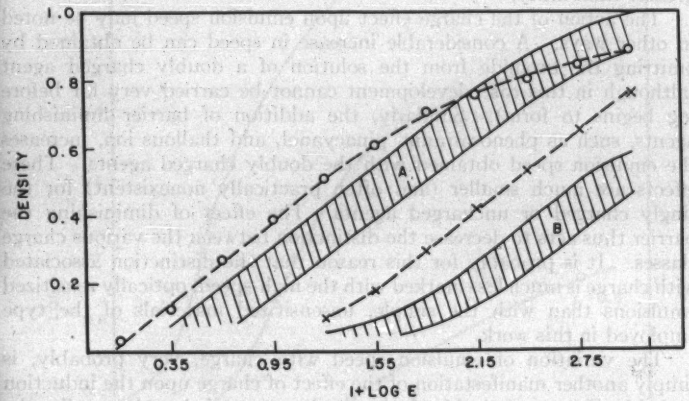


FIG. 2. Dependence of speed upon charge. Curves for singly charged catechol, *p*-aminophenol, methyl-*p*-aminophenol, dimethyl-*p*-aminophenol and *p*-aminocarvacrol fall in zone "A"; those for doubly charged hydroquinone, chlorohydroquinone, bromohydroquinone, ascorbic acid and *p*-hydroxyphenylglycine fall in zone "B." Curve –○–○– represents diaminodurene; curve –×–×– represents methyl-*p*-aminophenol monosulfonate.

previously found to give low maximum gamma and speed (hydroxylamine and *p*-aminophenylglycine) were now found to give, on partial development, markedly lower emulsion speeds than the more typical singly charged agents. The curves fell in with those of the group corresponding to the doubly charged agents. *In no case, however, has a doubly charged agent been found which, in the early stages of development, gives an emulsion speed comparable to those obtained with the typical singly charged agents.*

Under the conditions employed, the characteristic curves obtained for partial development by the uncharged agents often do not have well-defined straight-line portions. Accordingly, determinations of gamma are of lower accuracy and of less significance for these agents. The curves in most cases lie in about the same region as those of the typical singly charged agents. In most cases, however, the sulfite is not adequate to remove the oxidation products of the *p*-phenylenedi-

amine derivatives, and these products have an adverse effect upon development. In the one case examined where the oxidation product is without significant effect upon the course of development (diaminodurene), a somewhat higher speed was obtained than that given by the best of the singly charged agents. Even in this case the distinction is small. The really significant distinction appears in comparisons of uncharged or singly charged agents with those having a charge of greater than unity.

The action of the charge effect upon emulsion speed may be noted in other ways. A considerable increase in speed can be obtained by omitting the bromide from the solution of a doubly charged agent (although in this case development cannot be carried very far before fog begins to form). Similarly, the addition of barrier-diminishing agents, such as phenosafranin, pinacyanol, and thallous ion, increases the emulsion speed obtained with the doubly charged agents. These effects are much smaller (and often practically nonexistent) for the singly charged or uncharged agents. The effect of diminishing the barrier thus acts to decrease the distinction between the various charge classes. It is probably for this reason that the distinction associated with charge is much less marked with the high-speed, optically sensitized emulsions than with the simple, unsensitized materials of the type employed in this work.

The variation of emulsion speed with charge, very probably, is simply another manifestation of the effect of charge upon the induction period.[2] On a statistical basis, it may be assumed that the smaller the exposure, the smaller will be the localized drop in the bromide barrier resulting from latent-image formation,[3] and hence the larger will be the relative induction period obtained with charged developers. Thus, *for intermediate degrees of development*, the charge is a factor *in addition to* the normal considerations of size and configuration of the latent-image nuclei in determining the dependence of development upon exposure. The charge effect discriminates in favor of the higher exposures. If the times of development by two agents, one charged and the other uncharged, are adjusted so that equal densities are obtained at some high-exposure step, the lower steps should yield smaller densities with the charged agent than with the uncharged agent, and the characteristic curve for the charged agent accordingly should show a higher *gamma* value. This result is the one actually obtained in practice in the absence of complicating factors, as shown in Figure 1. As is generally true when the charge is involved, the differentiation is marked between the singly and the doubly charged agents. The addition of bromide to the developing solution increases the differentiation, while the addition of phenosafranin, pinacyanol, and thallous ion decreases it.

[2] T. H. James, *J. Frank. Inst.* 240, 83 (1945).
[3] J. E. de Langhe, *Z. wiss. Phot.* 35, 201 (1936).

INFLUENCE OF SULFITE UPON THE EMULSION SPEED.

Sodium sulfite, in sufficiently large amounts, markedly increases the emulsion speed obtained by partial development in a solution of a doubly charged agent. The primary effect, free from complications caused by oxidation products, can be studied conveniently with iso-ascorbic acid as the developing agent. Figure 3 gives some results

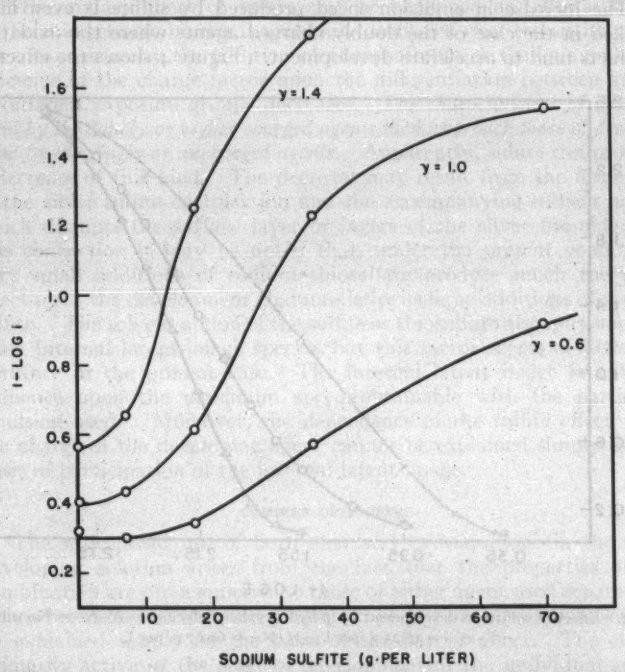


FIG. 3. Effect of sulfite on speed obtained with iso-ascorbic acid developer. Composition of solution: iso-ascorbic acid, 0.005M; excess KOH, 0.04M; KBr, 0.00667M; Na_2SO_3, xM; Na_2SO_4, $(0.55 - x)M$.

obtained with this agent. Here, the logarithm of the inertia speed (expressed in terms of $1 - \log i$) is plotted against the sulfite content of the developer. Curves relating to three different gamma values are given. The total salt concentration of the developer was held constant throughout the series by appropriate additions of sodium sulfate. Control experiments showed that neutral salt itself produced some increase in speed, but the increase was quite small in comparison with that produced by the sulfite.

Entirely similar results were obtained with a potassium ferro-oxalate developer at pH = 8.4–8.8. This developer contained a high excess of oxalate. The ferro-oxalate ion color was dominant, and the complex ion was the active developing agent. Potassium sulfite was used in place of the sodium salt, because of the relatively low solubility of sodium oxalate.

The increase in emulsion speed produced by sulfite is even more marked in the case of the doubly charged agents where the oxidation products tend to accelerate development. Figure 4 shows the effect of

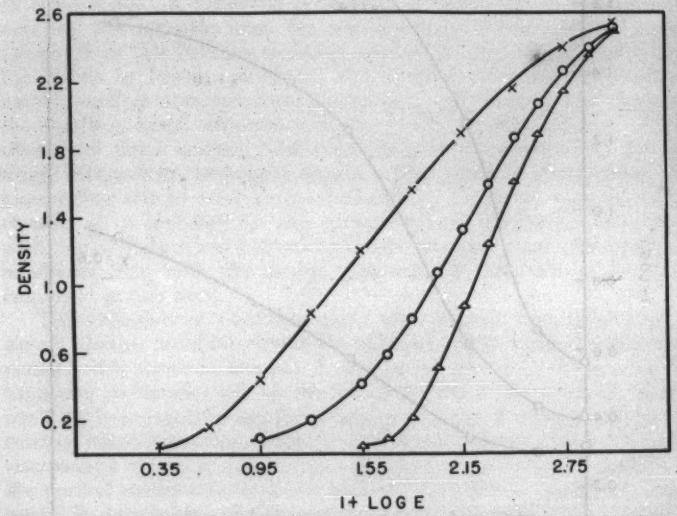


FIG. 4. Effect of sulfite on development by *p*-hydroxyphenylglycine. –Δ–Δ– = No sulfite; – o – o – = 16.25 g. per l.; –×–×– = 66.67 g. per l.

sulfite upon the shape of the characteristic curves obtained with *p*-hydroxyphenylglycine (glycin). The curves represent developers containing no sulfite, 16.25 g. per liter, and 66.67 g. per liter. Development times were chosen such that equal densities were obtained for the first exposure step (log $E = 2.05$). The effect is even more pronounced in the case of hydroquinone.[4] The part played by the oxidation product in this connection has been explained already.[4]

Sulfite had considerably less effect upon emulsion speed in the case of the singly charged agents. Some increase in speed was observed when sulfite was added to solutions of certain uncharged agents, but

[4] T. H. James, Part III, *J. Frank. Inst.*, 240, 229 (1945).

only in those cases where reaction between the sulfite and the oxidation product occurred readily. The increase in these experiments probably resulted simply from the removal of the detrimental oxidation product. No increase whatsoever was obtained when sulfite was added to a solution of diaminodurene. Thus, the familiar pattern of the charge effect is repeated once more.

The primary effect of sulfite may be explained as follows: When the bromide charge barrier is uniformly decreased over the entire range of exposures (as by the addition of phenosafranin or thallous ion), the influence of the charge factor upon the differentiation between grains in different exposure groups decreases. *The characteristics of development by the doubly or higher charged agents then approach those of development by the singly or uncharged agents.* Apparently, sulfite can produce a decrease of this kind. The decrease may result from the formation of the silver sulfite complex ion and the accompanying solvent action which disrupts the surface layer or layers of the silver bromide. In this connection it may be noted that, under the present conditions, very small additions of sodium thiosulfate produce much the same effect upon the development characteristics as large additions of sodium sulfite. The solvent action of the sulfite or thiosulfate also may uncover some internal latent-image specks, but this factor seems of little importance in the present case. The internal latent image is without influence upon the maximum speed obtainable with the particular emulsion used.[1] Moreover, the dependence of the sulfite effect upon the charge of the developing agent cannot be explained simply on the basis of participation of the internal latent image.

COMPLEX DEVELOPERS.

The widespread use of both elon and hydroquinone in the same developing solution arises from the fact that the properties of the combination are often superior to those of either agent used separately.[5] It has been shown [6] that the general properties of the combination can be explained simply on the basis of the barrier effect. The elon is primarily active at the start of development of the individual grains. As development proceeds, the bromide barrier decreases and the hydroquinone takes a more prominent part in the reaction. In solutions of fairly high pH, such as D-19, the hydroquinone assumes the dominant rôle in the continuation of development of the grains.

If the preceding explanation is correct, other pairs of developing agents should exist which would show the same type of superiority over the properties of the individuals as that encountered in the elon-hydroquinone combination. The requirements are that the two agents

[5] Cf. C. E. K. Mees, "Theory of the Photographic Process," p. 367 ff. The Macmillan Co., New York, 1942.

[6] T. H. James, *J. Phot. Soc. Am.*, 9, 62 (1943).

differ in charge, and that the individual rates be such that the agent of low charge develops more rapidly in the induction period region, but the agent of higher charge develops more rapidly beyond that region. Compliance with the latter requirement is often just a matter of determining the proper concentrations and pH range. A further requirement seems to be that the oxidation product of the agent of low charge must not actively interfere with the development process. The use of sulfite in the developer effectively eliminates the oxidation product of elon, but does not sufficiently remove the oxidation products of a number of the members of the *p*-phenylenediamine series.

The present explanation of the nonadditive properties of the elon-hydroquinone combination and other similar combinations attributes the phenomenon primarily to a change in the relative rates of reaction of the partners on passing beyond the induction period region. This is in agreement with the views of Reinders and Beukers.[7] Another, older explanation assumes the formation of an active complex between the partners, and attributes special developing properties to that complex.[8,9] In this respect, the behavior of the combination of hydroxylamine and sodium hydroquinone monosulfonate is of importance, since the formation of an active complex between these two agents is highly improbable. The data in Table I show that the combination

TABLE I

Development by the Hydroxylamine-Hydroquinone Monosulfonate System. Na_2SO_3, 0.125 M; pH = 12.6; Temp. 20° C.

Exposure log *E*	Dev. time (min.)	Densities obtained with		
		0.01 *M* hydroquinone monosulfonate	0.00167 *M* NH_2OH	Combination of the two
1.15	12	0.00	0.01	0.15
1.15	20	.05	.03	.83
1.75	12	.01	.04	.34
1.75	20	.16	.15	1.42
1.75	30	.84	—	—

exhibits the typical superadditive behavior with respect to reaction rates. Definite, although less marked, superadditive properties have been observed in the methyl-*p*-aminophenol—methyl-*p*-aminophenol sulfonate combination. Again, the formation of a complex is highly improbable.

ROCHESTER, N. Y.,
May 31, 1945.

[7] W. Reinders and M. C. F. Beukers, *Phot. J.*, 74, 78 (1934).
[8] A. and L. Lumière and A. Seyewetz, *Phot. Korr.*, 62, 182 (1926).
[9] V. Shiberstoff, *Kino Photo Ind., U.S.S.R.*, 2, 141 (1933).

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

FLIGHT SIMILITUDE TESTS OF RADIOSONDES.

Accurate weather forecasting requires a knowledge of the weather conditions in the air up to some 50,000 to 55,000 feet. In past years it was the custom to make routine airplane flights to secure a partial knowledge of these conditions. These flights were costly and rarely reached 35,000 feet.

The development of the radiosonde, a fairly cheap and light instrument, which can be carried to heights of 60,000 to 80,000 feet by means of small balloons, has made it possible to learn of these conditions much more frequently, more accurately, and more cheaply.

The elements of the instruments which measure air temperature, pressure, and humidity, control the signals from a radio transmitter. These signals are received and recorded on the ground in a form easily converted to readings of temperature, pressure, and humidity. Temperatures below − 60° C. (− 76° F.) may be recorded in the altitude range of 60,000 to 80,000 feet.

Acceptance tests are being made at the Bureau on random samples of radiosondes purchased by the Navy Department. A flight similitude test to obtain the performance of the sondes in measuring pressure and temperature is an important part of the work. In this test the radiosonde is subjected simultaneously to decreasing air pressure and temperature at controlled, specified rates that correspond to average conditions in an actual ascent. The signals are received on a recorder similar to that used in service and on an auxiliary recorder. In addition, the actual pressures and temperatures in the chamber, measured by standard instruments, are recorded so that the errors of the radiosonde indications can be determined.

The pressure chamber is a cast iron cylinder with pressure taps and electrical connections, and is equipped with a fan operated by an external electric motor to ventilate the sonde during the test at a rate corresponding to the ventilation in actual flight. The pressure chamber is installed in a liquid bath to which pulverized dry ice is added as required to maintain the specified rate of decrease of temperature in the pressure chamber. The temperature of the radiosonde temperature element is measured by thermocouples; the pressure by a precise, calibrated aneroid barometer.

* Communicated by the Director.

ENGINE PERFORMANCE OF SUBSTITUTE MOTOR FUELS.

In the course of the investigation of substitute motor fuels, conducted by the Bureau for the Foreign Economic Administration and referred to in *Technical News Bulletin 339* (July 1945), studies have been made of many phases of fuel performance in both test and stock engines. The substitute fuels used in the investigation included charcoal gas; shale oil; naptha; and vegetable fermentation products such as ethyl alcohol, acetone, butanol, and other materials. The work on liquid fuels covered studies of the volatility of fuels and blends, rate of evaporation loss, effect on pump diaphragms, starting, warm-up, vapor lock, manifold distribution, knock rating, power, economy, spark advance, oil consumption and dilution, engine deposits, cylinder wear, and exhaust composition.

As explained in an article that Donald B. Brooks has prepared for publication in "Automotive Industries" this investigation showed that use of substitute liquid fuels would not result in increased wear or corrosion of the engine or fuel system components. The maximum power developed with these liquid fuels is within a few per cent. of that developed with gasoline, but the fuel mileages are poorer in proportion to the lower heats of combustion of the substitute fuels. Engines especially designed to utilize the higher antiknock properties of these fuels would reduce the difference in fuel mileages. Where liquid fuels are not available, gas generated from charcoal or other solids can be used in automotive equipment, but only at the sacrifice of nearly half of the power.

REVIVIFICATION OF BONE CHAR.

A report is now being prepared on the recent study of three industrial units for the revivification of bone char. It has been necessary to supplement the plant data by some pertinent laboratory determinations. About 60 samples of bone chars were taken during this investigation, and these are being analyzed for their content of carbon and hydrogen, and for the total loss-upon-ignition of the acid-washed residues. The pH of aqueous extracts of these chars is likewise being obtained. In order to supplement the gas-analyses data already collected, some additional information is being obtained on the composition of the gases expelled upon heating a given sample of service char under three conditions: (1) The original revivified char alone, (2) the original revivified char plus a small amount of sucrose, and (3) the original revivified char plus blackstrap molasses, followed by thorough water-washing.

The three types of furnaces available for this investigation differed widely in their modes of heat transfer. In the conventional kiln the products of combustion of the fuel were in contact with the outside

walls of the retorts. The heat flow, therefore, was through the metal walls into the inclosed cylinder of char. The gas space in contact with the char was simply that which exists between char particles. On the other hand, the gaseous products of combustion of the fuel in the Herreshoff furnace were in direct contact with the bone char. The heat flow in this case was from the higher temperatures of the gas to the slowly moving bed of char beneath. The gaseous products of the bone char were diluted greatly by the gaseous combustion products of the fuel and the large volume of recirculated gases. The modified Weinrich decarbonizer presented the third example. The rotating drum of the decarbonizer was externally heated. The char within the drum was lifted and cascaded to the bottom from where it was again lifted. Heat flowed through the metal walls of the drum into the bed of char, and because of the intimate contact with the air within the drum, the air assumed the temperature of the char. Therefore, only a small temperature gradient existed within a given cross-section.

In the completed report, an attempt will be made to present a discussion of these three pieces of equipment to bring out their individual characteristics. It is expected that the data obtained in this work will lead to the design and building of an improved pilot plant for the treatment of bone char.

VAPOR SEALS FOR HOUSE WALLS.

The Bureau's investigation of vapor barriers for walls of houses has been extended to include the possibilities of combining vapor resistance with decorative features in interior finish so as to provide barriers that would be continuously accessible for inspection, and to reduce fire hazards.

Samples of wall papers applied in the conventional manner allowed 1400 grams of moisture per square meter per day to pass through the wall, whereas samples of the same paper applied with a vapor-resistant adhesive reduced the passage of moisture vapor 100 fold. Samples of wall papers having a vapor-resistant coating on their face, and applied in the conventional manner also reduced the passage of moisture vapor 100 fold. Unfortunately, no vapor-resistant adhesive or coating has been found that does not disfigure wall paper. Vapor-resistant plastic sheetings, though difficult to apply, were found in other respects to be very satisfactory. Paint films and varnishes with and without metal powders were easy to apply but gave variable results. Good aluminum, copper, and other metal foils were in most cases impervious to moisture vapor. Hence, particular attention is being given to the possible use of foil on the interior surfaces as a base for decorative finishes.

SPLICES IN REINFORCING BARS FOR CONCRETE.

It is frequently necessary to splice reinforcing bars at various points in order to preserve the continuity of concrete structures. A common method of splicing is to provide for a sufficient overlap of the reinforcement to permit the transfer of stress from one bar to the other by means of a bond between the bar and the surrounding concrete.

To obtain information on the behavior of splices in beam reinforcement a series of tests was conducted. The findings are presented by Ralph Kluge and Edward C. Tuma in the *Journal of Research* for September (RP1669).

The principal objectives of the investigation were: To determine the distribution of stress along the lapped bars of the splice and the accompanying bar slip; to compare the effectiveness of two methods of lap splicing, one with the lapped bars in contact with each other; and lastly to compare the behavior of two distinctly different types of reinforcing bars in lap splices of various lengths.

The tests showed that a length of splice 30 or more times the bar diameter was sufficient to develop the yield strength of the bar. Also, that the bond stresses developed within the first few inches of the free end of the bars, lapped 30 or more bar diameters, was independent of the length of the splice but was roughly proportional, within certain limits, to the tensile stress in the reinforcement outside the region of the splice. Contrary to expectations and also to general opinion, the method of lap splicing had no appreciable effect on the behavior or strength of the splice. A substantial difference was observed, however, in the general behavior and maximum bond resistance of the two types of bars.

MINIMUM DESIGN LOADS IN BUILDINGS.

With the completion of American Standard Building Code Requirements for Minimum Design Loads in Buildings and Other Structures, another standard has been added to the series that is being produced under the procedure of the American Standards Association. In this case, the Bureau acted as sole sponsor of the project. The standard contains recommendations for assumptions regarding dead and live loads, wind pressures, and earthquake forces. Some of these assumptions hold good in any part of the country, whereas others must vary with local conditions. To deal with this situation, the minimum requirements suggested in the case of snow, wind, and earthquakes are supplemented by information supplied in an appendix; this should be useful to local code committees in making necessary adjustments. The appendix also contains tables of weight of materials and of loads caused by different kinds of occupancies, and presents other information intended to assist in the application of the code requirements.

The standard introduces a new treatment of reduction of live loads, based on an investigation of actual loads undertaken by the Public Buildings Administration. It also provides for wind pressures acting outward on walls and roofs, as well as inward. This recognition of outward pressures is in accordance with the known facts, but is a feature not covered by most existing building codes. In the preparation of the standard, use was made of a great deal of technical information available in various divisions of the Bureau.

World's Largest Gas Engine Plant, Rated at 78,000 KW., Powers Aluminum Works.—(*Power*, Vol. 88, No. 11.) From the time when Charles Martin Hall first made aluminum a commercial reality, direct current electrical energy has been a basic ingredient, virtually as important as the aluminum oxide from which the metal is made. Heretofore, power demands have been met by high voltage alternating current hydroelectric stations with motor generator sets or mercury arc rectifiers for conversion to direct current. However, when Aluminum Co. of America started design of a Defense Plant Corp. works at Jones Mills, Ark., near large ore sources at Bauxite, lack of available hydro power and war urgency turned attention to other sources, notably steam power where the advent of higher pressures and temperatures had raised efficiency to new levels. Out of preliminary studies grew plans for a modern plant with three 35,000 KW. turbine generators.

At this stage, difficulty in securing turbine allocations necessitated examining other possibilities for at least part of the load. Studies revealed that internal combustion engines driving direct current generators compared favorably on an economic basis with alternating current turbine generators and conversion equipment. Meanwhile, formation of the Southwest Power Pool made transmission line power available.

As completed, the Jones Mills Works consists of four pot lines, with a carbon anode plant and auxiliary facilities. Transmission line power supplies two pot lines; gas engine generators supply the other two. Engine plant capacity totals 78,000 KW. in 68 units; 18 Nordberg gas diesels (40,500 KW.) and 50 Cooper Bessemer convertible gas engines (37,500 KW.). The latter are 1165 hp. 10 cyl. arranged 60 deg. V 14 × 14 in. 350 rpm. 2 cycle units, the former are 9 cyl. 21½ × 29 in. 225 rpm. 2 cycle 3600 hp. of the dual fuel or gas burning diesel type.

Low cost natural gas fuel comes by pipeline, from northern Louisiana and nearby southern Arkansas. Its heating value is about 980 B.T.U. per cu. ft.

Electrical circuits are divided into two parts: direct current for pot lines and alternating current for plant auxiliaries. On the direct current side, individual positive and ground circuit breakers, of high speed open design, connect each engine generator to plant busses that connect, in turn, through an outdoor bus structure, with silver conductors, to pot lines. Normally, each engine plant serves an individual pot line; mercury arc rectifier equipment serves the two remaining pot lines from the transmission lines.

R. H. O.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Research Projects.—ELLICE MCDONALD, Director. Every organization has different aims and the direction is influenced by the purpose for which it has been begun. For example chemical research naturally has to do with the production, adaptation and development for use of chemical substances. If the purpose lies in one phase of science, the organization is fortunate, for there are then fewer variables to consider. Physics may be taken as another example where at present it is but little taken over to aid in other branches of science. It is only now beginning to be drawn over into other areas to illuminate special fields of investigation.

An attempt has been made to classify branches of science as to basic or otherwise. Not knowing exactly what "basic" means in this connection, it seems better to use the term exact or fundamental for, after all, the division between the branches of science depends upon the possibility of quantitative measurements and the accumulation of measured facts capable of confirmation. And after all, all phases of science are one in that they are the expression of the nature and reactions of matter: they depend upon the forces of nature and the totality of its laws.

Biochemistry and biology are not so fortunate as to be able to explain the mechanism of the action of the forces it studies. Life is still a mystery, both to the anatomists and the mechanists. Everywhere in biochemistry we come finally up against the profound mystery of life itself. For this reason there are many unknown variables and few criteria of accuracy. So biochemistry must involve many other phases of science, chemistry, physics, and cytology with all their ramifications. For this reason, the choice of research projects is difficult.

Like greatness, some inherit research projects, some achieve research projects and some have research projects thrust upon them. In some organizations the avenues of adventure to follow are more or less well defined. In the great new research center which Kettering has designed, the research will be to make better automobiles; and the motor car with some understanding of its mechanism is there to start with so that it is possible to plan research into improvements on an exact basis, better design, better motive power, better moving parts, always with the hope that some illuminating factor will evolve outside all plans but resulting from the atmosphere of research and the stimulation of the imagination.

This limits the problems a bit, although it is sure that Kettering would not agree to this statement; but in chemical research laboratories

there seems to be a wide scope as to what may be possible, so that the line to be followed cannot be so well defined. The great discoveries of chemistry and the enormous possibilities of future discovery of things useful to mankind stagger the imagination. The field is so large that the area upon which to work must be difficult to decide.

Biochemical research projects, therefore, must be more carefully chosen and scrutinized. The unbelievable advances of the last half century seem to offer almost boundless possibility. The human mind is inclined toward doubt and scepticism and toward an inherent fear of change which may upset things as they are. This seems to be the origin of doubt as to future discovery which is the unfolding of new things. Discovery is always a surprise and its form cannot be really anticipated. All we know is that discovery will occur and that it is more likely to occur at the present time than at any other period in the past.

The amount of past discovery makes more future discoveries so probable as to be almost a certainty. The curve of research success is still upwards and at a sharp angle, so that it would require a catastrophe to stop its upward slant. We cannot anticipate what form it will take: we can only surmise the phase in which it will appear. But who could have anticipated radio, television, the internal combustion engine: things that are taken as commonplace now. All we know is that based upon past accumulated and recorded experience, new discoveries will come.

What an adventure is this to the next generation. They will live in an atmosphere of intellectual stimulation which should energize them more than anything that has gone before and give them a continual uplift of spirit so that they will pass their lives in a glow of imagination and in a great sphere of activity. And won't they have fun!

But for the present, research projects must be chosen in relation to the present state of past discoveries. The advice of a professor at Massachusetts Institute of Technology to some of his students who asked advice as to their future course in life was, "First do what you want to do; if you can't do that do what you can; if you can't do the latter, try to make money." If "make money" is paraphrased into "do research along classical lines, follow and develop recognized procedures and confirm and extend known experiments," this advice will hold good for the selection of research projects.

BOOK REVIEWS.

INDUSTRIAL OIL AND FAT PRODUCTS, by Alton E. Bailey. 735 pages, tables, 15 × 24 cms. New York, Interscience Publishers Inc., 1945. Price $10.00.

Fats are at present the cheapest and most satisfactory source of long hydrocarbon chains which are definite in structure and reactive in nature. Such structural units are essential in the manufacture of a great variety of useful substances. While it is true that long hydrocarbon chains are also found in petroleum, and that methods may be eventually developed for modifying these chains to make them properly reactive, the fact remains that the preparation of fully satisfactory fat substitutes from petroleum remains far from accomplished. Furthermore, the world's reserves of cheap petroleum are not inexhaustible, and once exhausted cannot be replaced, whereas oils and fats may continue to be grown indefinitely.

This book is intended for use as a text on oil and fat technology. It is divided into four parts, the first of which is devoted to the nature of fats and oils. This discusses the structure and composition, the reactions of fats and fatty acids, the physical properties of fats and fatty acids and the role of fats in the diet of man. Part two covers raw materials for oil and fat products—sources, utilization, production and consumption of primary fats and oils, and composition and characteristics of the individual fats and oils. Under the heading of industrial utilization of fats and oils, treatment is given to the topics of cooking and salad oils, plastic shortening agents, butter and margarin, bakery products and confections, soap and other surface-active materials, paints and varnishes and miscellaneous oil and fat products. This part and that following are quite complete with the filling in of most of the gaps in the literature of oil and fat technology, with material published for the first time.

The last part of the work is devoted to unit processes in oil and fat technology. The specialized branch of fat technology—extraction of fats and oils is first taken up and this is followed by references to refining and bleaching, deodorization, hydrogenation, soap making, fractionation, fat splitting, polymerization etc. in all, there are 710 pages of text, a large amount of material, well classified, and presented in a connected sequence. The work is a timely addition to the literature of a field which has experienced a rapid advance in the past few years.

R. H. OPPERMANN.

STUDIES IN BIOPHYSICS: THE CRITICAL TEMPERATURE OF SERUM (56°), by Lecomte Du Noüy. 184 pages, tables and illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1945. Price $3.50.

This book reveals new facts brought to light from experimental work mainly done at the Pasteur Institute and which is closely related to the fundamental problems on the nature of alexin, of the sensitizer, and of the mechanisms of the action of the complement. Descriptions are given of twelve phenomena, ten of which begin around 56 degrees when the serum has been heated for ten minutes in a sealed vessel; they are increase of viscosity, of rotatory power, of rotatory dispersion, of absorption of light, of scattering of light, of the depolarization factor, of electric resistivity, of concentration in hydrogen ions, a decrease of the power of fixation of ether, modification of the stability and of the sedimentation of the globulous, of the interfacial tention, and of the absorption curve in the ultraviolet. In undiluted serum, the increase of the concentration of hydrogen ions generally starts around 60 degrees, and the modifications in the absorption curve are clear at 65 degrees.

It is claimed that each of the new facts revealed can be considered as a starting point for other researches in the same direction or in a direction more specifically chemical. Interpretation of the new facts has been made by means of a hypothesis concerning the structure and behavior of the complex lipido-protenic "molecule" of the serum which has been useful in planning experiments.

R. H. OPPERMANN.

ELEMENTARY ELECTRIC-CIRCUIT THEORY, by Richard H. Frazier. 434 pages, illustrations, 14 × 22 cms. New York and London, McGraw-Hill Book Co., Inc., 1945. Price $4.00.

Now more than ever before is it necessary for the electrical engineer to have a broader view of the happenings in the various parts of the frequency spectrum in order to appreciate developments that influence professional practice in his special branch. The book at hand is a text on the elements of electric circuit theory which stresses those broad factors basic to the interests of the power engineer as well as electronics and communication engineers.

The treatment begins with electric circuit definitions and concepts where some correlation between electric circuit abstractions and corresponding physical apparatus is given, and proceeds through resistance networks and basic alternating current concepts. At this point the use of complex algebra is explained and this provides a basis for the development of the many alternating current relations which follow. After this the subject of resistance networks is recalled by substitution of complex quantities for scalars and redeveloped in some instances to the extent necessary to clarify the application to impedance networks. Subsequently the elementary principles of analysis of nonsinusoidal waves by means of Fourier series are explained, the elementary relations of polyphase networks are presented with emphasis on the through phase network, and finally the classical method of transient analysis is outlined. There are appendixes with useful information and tables, and a subject index.

The trend is logical and well connected. The problems used for illustrations and those for exercise deserve special note for their variety and practicability. The work is good for the classroom, for refresher purposes, and as a reference. R. H. OPPERMANN.

THE CHEMISTRY OF LEATHER MANUFACTURE, by George D. McLaughlin and Edwin R. Theis. 800 pages, tables and illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1945. Price $10.00.

A marked change has occurred in the leather industry during the past generation. Thirty years ago there were approximately 740 tanneries in the United States compared with 440 today, although the present volume of leather produced is considerably greater. This centralization of production went hand in hand with the growth and application of tanning science, and the unscientific tanner was unable to survive. The present tanning generation faces an even greater test, because many leather substitute materials have appeared and others are being studied by competent scientific minds. The leather industry is becoming increasingly aware that empirical methods must be replaced by scientifically sound procedures.

The purpose of this volume is to discuss the theoretical phases of tanning, by summarizing and appraising our present scientific knowledge. The volume is in its third edition, the first two had the late John Arthur Wilson as the author and were published in 1923 and 1929 respectively. Certain parts of the second edition were omitted in the third in order to make way for expansion of some subjects. The opening chapter on histology of skin is one of these and only a very general outline is given, sufficient, however, to serve as a background for a summary of available knowledge and to present a picture of the skin's composition and chemical characteristics. Skin pigments and the more important skin enzymes are included. This is followed by a discussion of the acid or base fixation of proteins in which the main interest is in the available amino and carboxyl groups of the polypeptide chain, and the significance of the boil test in determining facts relating to the structures and the structural stability of the skin collagen. Microörganisms and their behavior, and curing are subsequently taken up with soaking, liming and dexilation, bating, pickling. A great part of the work is devoted to tanning, aldehyde, quinone, chrome vegetable, iron and alum tanning. A chapter is given to syntans and their classification. The subjects of miscellaneous tannages, neutralizing, flat-liquoring, oiling and stuffing leather, and physical testing methods complete the book. There are subject and author indexes in the back.

This American Chemical Society Monograph is a worthy volume for placing alongside other such monographs. It is an exhaustive survey of the literature with the contributions of the best of knowledge of the subject. R. H. OPPERMANN.